Closure Puzzles

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1 Intro

If one statement or claim implies another, and the first is clearly true, then one would expect the second to be clearly true too. Controversy should not erupt between the premises and the conclusion of a valid argument. [1] And yet sometimes the weaker statement does in fact seem, if not controversial, then at least harder to know than the stronger one. Here are a few examples.

1. (Carnap) The number of tiles on this floor is composite. So there are numbers.
2. (Nozick) I have a hand. I am not a handless BIV being fed hand-impressions.
3. (Dretske) That's a zebra. So it's not a cleverly disguised mule.
4. (Cohen) That is red; for it looks red. Hence my color vision is on this occasion accurate.
5. (Kripke) I locked the door. Any evidence there may be that I didn't lock the door is misleading.
6. (Field) My theory has axioms A_1, A_2.... It must be consistent, because each A_k is true.

I am not saying these cases are all the same; they're clearly not. But there's certainly some kind of family resemblance. What have people had to say about them?

Nozick suggested an explanation of (2). You know something only if your belief tracks the facts. You know that p only if your belief that p is sensitive to whether p is true; had it not been true, you would have noticed. That you would have noticed if you didn't have a hand doesn't mean you would have noticed if you were a handless brain in a vat. That you would have noticed had the number of tiles been prime doesn't mean you would have noticed had there not been numbers in the first place. Sensitivity to p does not carry with it sensitivity to q just because p implies q.

Kripke objected to Nozick's theory that if you can't know things on the basis of validly deducing them from other things you know, then intellectual life as we know it is over and we should just throw in the towel. (Fallacy of logical deduction.) He also gave some examples to show that closure on Nozick's theory fails much more dramatically than he seems to realize; I can know it's a red barn (because if it hadn't been it would have been green which I'd notice) without knowing it's a barn (because if it hadn't been it would have been a convincing fake barn which I wouldn't have noticed. This and similar objections led to people giving up on Nozick and looking for alternatives that respected closure. (This may have been premature. Later I'll suggest a Nozick-like theory that respects closure.)

[1] Assuming, anyway, that the argument's validity is itself clear and uncontroversial.

Another explanation of what is going on in some of these cases is contextualism. This is the view that "knowing" becomes a different and harder thing en route from *I have a hand* to *I am not a BIV*. The seeming paradox of knowing the first without knowing the second becomes the non-paradox of knowing$_1$ that I have a hand while not knowing$_2$ that I am not a brain in a vat. Contextualism respects closure because I don't know$_2$ that I have a hand either. (Contextualism is normally seen as opposed to Nozick's tracking theory, but the Nozick-like theory I'll be suggesting later is a contextualist tracking theory.)

Then finally there is Carnap's explanation of what is going on in case (1). Let's remind ourselves how the problem arises in the numerical case. Let α be a "specific" numerical claim, of the sort we'd encounter in pure or applied arithmetic. And let ω be a general claim to the effect that numbers exist—or, since the numbers stand or fall together, that there are *the* numbers, there is the number *system*. The paradox is that the following three propositions look highly plausible:

1. α entails ω.
2. It is clear that α.
3. It is controversial whether ω.

They look plausible, but, assuming anyway that uncontroversiality is preserved under obvious entailment, the three propositions cannot be true together. Something has got to give.

Carnap's solution is this. When we run through the argument in our heads—*The number of Fs is prime*, therefore, *There is such a thing as the number of Fs*, therefore *Numbers exist*—it feels as though something changes—some cognitive switch is pulled—en route from α to ω, in a way that bears on our judgments of clear vs controversial. When we say, or reflect internally, that *The number of Fs is prime*, we are addressing one sort of question: an internal question, whose answer is to be found by following the rules of the number-framework.

But when we get to *Numbers exist*, or *The number system exists*, things are different. We are unlikely to be asking an internal question here, because so understood the question would be silly. *Obviously* the number framework instructs us to say there are numbers, and speakers generally try to avoid stating the obvious. A system of rules making "There are numbers" *unproblematically* assertible is a system of rules in need of external validation, or the opposite. Are the rules right to counsel acceptance of "There are Xs"? It is no good consulting the framework for the answer; we know what *it* says. No, the existence of Xs will have to queried from a position outside the X-framework.

Now, Carnap does not entirely reject this line of thought. He respects the ambition to cast judgment on the framework from without. He just thinks we have a wrong idea of what is coherently possible here. How can an external deployment of "There are Xs" mean anything, when by definition it floats free of the rules from which meaning comes? Charity thus requires us to substitute a different question. There are, fortunately, meaningful questions in the vicinity. These are questions that mention "X" rather than using it. One is the practical question: should we adopt a framework requiring us to use it like so? This is Carnap's candidate for the role of the external question;

The problem with this from our point of view is that it fails to his failure to explain why the external question should seem so philosophically controversial. The external question as he construes is not controversial in the least.. Show me someone who doubts that we should retain the number framework!

Another problem is empirical implausibility. Look what the framework theory says: Words depend for their meaning on analytically valid rules of assertion and inference. The rules completely determine proper usage modulo a given set of observation reports. Resistance is futile; all it achieves is to change the subject. There is no communicating with those who defy the rules, because they do not speak our language.

If twentieth century philosophy achieved anything, it was to show that this is not how natural language works. No rule of assertion or inference can lay claim to being analytically immune from doubt in the way that Carnap imagines. This is the moral, for instance, of Burge's "Arthur," who maintains that sofas are not pieces of furniture as on the usual theory but objects of religious veneration ([**?**])—he does not mean a different thing than us by the word "sofa," rather he has a different view of what sofas are.

If someone can mean the usual thing by "sofas" despite questioning their status as furniture (or the usual thing by "if" despite wondering about modus ponens), then surely someone can mean the usual thing by "number" despite wondering whether there are any. Communication is possible across all kinds of doctrinal divides—even when the doctrines are the kind considered "meaning-constituting" by those who go in for that sort of thing. Carnap's theory of frameworks erects barriers to communication that are empirically speaking just not there.

2 Internal/External

Back to the drawing board, then. Some cognitive switch is pulled en route from α to ω. But so far we have only succeeded in putting a label on it, or rather two labels: α purports to answer an"internal" question while ω addresses an "external" question. What the labels signify is still unclear. It might help to look back at how Carnap introduces the internal/external distinction in the first place, before he decides that internal questions are theoretical ("cognitive") and external questions are practical:

> there are, first, questions of the existence of certain entities ... within the framework; we call them internal questions; and second, questions concerning the existence or reality of the system of entities as a whole, called external questions ([**?**])

The idea of standing back from specific numerical questions and pondering the system of numbers as a whole is enormously intuitive. But commentators have had trouble with the idea, and it seems to play no further role in Carnap's thinking. Could it be that Carnap proposes to reconstrue seeming concerns about the number system qua system of entities as "really" concerns about the number system qua system of linguistic rules? I hope not; that is a pretty dramatic reconstrual. It might have its place in a rational reconstruction of our ontological thinking but not in a sympathetic assessment of that thinking.

Why not take the distinction as Carnap originally frames it at face value? Sometimes we wonder "about the existence or reality of the system of entities [the number system in this case] as a whole.". That is the external question. Other times we *presume* the system is there and wonder about what is specifically going on in it. To run it the other way around, asking about numbers "within the framework" is asking about them presupposing the number system, that is, taking for granted that it exists.[2] Asking about the system of entities as a whole is asking whether the presupposition is correct. Call that the *presuppositional* account of internal/external.

[2] An analogous contrast might be this: one can wonder whether a visually presented scene is "real" or just, say, moving images on a screen; but one can also, presuming the scene is real, wonder whether the cat is going to catch up to the mouse it is chasing.

The presuppositional account preserves the part of Carnap's story that worked best, the part about why it is hard to hear *There are numbers* as addressing an internal question. Wondering about the existence of numbers in an internal vein would be wondering whether, on the supposition that there are numbers, there are numbers. You'll recall that Russell made a lot of the fact that King George V, when he was wondering whether Scott was the author of Waverly, could hardly have been wondering whether Scott was Scott. I would say in a similar spirit that someone wondering about the existence of numbers can hardly be wondering whether numbers exist, supposing they exist. That is why we can easily hear *The number of dragons is* 0 as an internal remark but cannot easily hear *There are numbers* as an internal remark.

One problem with Carnap's account, recall, is the semantic wall it put up between insiders, operating within the framework, and outsiders wondering if framework rules might be mistaken. There might seem to be a similar problem here. Suppose I say that the number of dragons is 0 (call that S), taking it for granted that there are numbers (presupposing that π). Not everyone will join with me in taking it for granted that π; I may myself in philosophical moments not want to assume that π. What is my remark supposed to mean to those unwilling to grant even for discussion's sake that there are such things as numbers? It is not as though I expect my remark to fall on deaf ears in such cases. No, I mean to be saying something to the non-π-presupposers too, e.g, that there aren't any dragons. An account is needed of how communication is possible between π-presupposers and everyone else. How is it possible for outsiders to understand and perhaps even agree with what insiders are saying?

This will take some explaining, but the basic idea is as follows. Insiders are not asserting the full content of the sentence S coming out of their mouths; they are not representing that full content as true. What they are asserting is *the part of S that concerns the subject matter currently under discussion*. They may indeed have no way of characterizing this assertive content other than as the part of S concerning such and such a subject matter. What outsiders understand and agree with is the part of S just described.

3 Truth and aboutness

The idea of "truth about a specific subject matter" has a history in philosophy. Nelson Goodman and Joseph Ullian, in a 1977 paper called "Truth about Jones" ([**?**]), tell a pertinent story. Jones is on trial for murder and Falstaff is the chief witness for the defense. Jones's attorney concedes there is a problem with Falstaff's testimony. It is false. That would seem to be a fairly serious problem, but the attorney (Lupoli, he's called) thinks he sees a way out. The testimony was about his client Jones, no getting around that. And it was false, no getting around that either. But, Lupoli insists, the testimony was *partly true*: it was true in what it said about Jones.

The judge rejects this as nonsense and shuts down the proceedings, threatening Lupoli with contempt unless he can explain what he could possibly have meant by false and about Jones yet not false about Jones. Goodman and Ullian try to give Lupoli a hand with this, but the judge as I understand it was not impressed. I propose to give it another try, for Lupoli is surely on to something when he suggests that a false statement can nevertheless be partly true. Here is the naivest idea possible about partial truth.

(1) A hypothesis is *partly true* to the extent that it has parts that are (wholly) true.

But what is meant by *part* of a hypothesis? The naivest possible idea about that is

(2) One hypothesis is *part of*, or *included in*, another iff it is implied by the other.

The naivest possible idea about partial truth is correct, I think; something is partly true to the extent it has parts that are wholly true. But the naivest possible idea about parthood (inclusion) is questionable.

A paradigm of inclusion is the relation that typical conjunctions bear to their conjuncts–the relation *snow is white and grass is green* bears, for example, to *snow is white*. A paradigm of non-inclusion is the relation disjuncts bear to disjunctions; snow is white does not have snow is white or grass is green as a part. This is despite the fact that disjuncts imply disjunctions every bit as much as conjunctions imply conjuncts. On the face of it, then, there is more to inclusion than implication.

Good, but what is the missing ingredient? What is the X in *Parthood = implication + X*? An answer to this is implicit in the Jones story. Falstaff's testimony is partly true because *the part that concerned Jones* is true. A statement's parts are identified by looking for implications *whose subject matter is part of the subject matter of the original statement.* For B to be part of A involves, in addition to A implying B, that B's subject matter is part of A's subject matter. Conversely the reason A's implications are sometimes not part of A is that they bring in new subject matter. *Snow is cold* does not include *Snow is cold or grass is green* because the latter brings in the matter of grass's color which is wholly absent from *Snow is cold.*

4 Subject matter

A notion of subject matter was developed by David Lewis in "Statements Partly About Observation," among other papers ([**?**, **?**]). Lewis thinks we should not always be looking for some part or portion of reality that a sentence is about. Some subject matters are portions of reality, e.g., **the 20th century**, but others, like **the number of stars** are not. Subject matters are not parts of reality but aspects of reality. And the best approach to identifying an aspect of reality the best approach is to ask when that aspect *changes* as we travel from world to world.

Suppose we think of worlds that are just alike in the relevant respect as equivalent to each other, and worlds that differ in the relevant respect as inequivalent. Then that equivalence relation, the one just described, is what for Lewis counts as a subject matter. So, **the number of stars** is the equivalence relation that groups worlds together iff they have the same number of stars and sets them apart iff they have different numbers of stars. More generally a subject matter $\mathbf{m}$ is the relation that groups worlds together iff they are exactly alike wrt $\mathbf{m}$. Alternatively we can think of subject matters as partitions of logical space putting worlds into the same cell iff they are relevantly alike. Each cell is a set-of-worlds proposition that specifies one of the ways things can be with respect to the relevant subject matter. E.g.,. one cell of **the number of stars** is the set of all worlds with thirty eight stars, aka the proposition that there are thirty eight stars.

One nice thing about Lewis's theory of subject matters is that it allows us to define all sorts of interesting relations on subject matters. Subject matters are orthogonal, for instance, if each cell of the one overlaps each cell of the other. The relation that interests us is *inclusion.* Lewis has a theory of this, too. He reasons as follows: it is harder for worlds to agree on a larger subject matter (**the 20th century**) than a smaller one (**the 1920s**). Turning this around, a subject matter is larger to the extent that it is harder for worlds to agree on it. So, thinking of $\mathbf{m}$ and $\mathbf{m}'$ as equivalence relations, $\mathbf{m}$ is bigger if whenever two worlds are $\mathbf{m}$-equivalent, they're also $\mathbf{m}'$-equivalent.

(3) $\mathbf{m}'$ is a *part* of $\mathbf{m}$ iff $\mathbf{m}$-equivalent worlds (on which $\mathbf{m}'$ is defined) are $\mathbf{m}'$-equivalent.

Thinking of **m** and **m′** as partitions, **m**is bigger, roughly if its cells are included in cells of **m′**. A subject matter is larger to the extent that its cells are smaller.

Now, Lewis has a theory of subject matters, but he offers no theory whatever of *sentential* subject matters: of what S's subject matter is for particular sentences S. We are told only that **m** is *a* subject matter of S iff S's truth-value "supervenes" on **m** in the sense that each proposition in **m** implies either that S is true or that S is false. But that gives S a huge range of subject matters ranging from, at the low end, the two-cell subject matter **whether S is true** , to, at the high end, the (presumably) one-world-per-cell subject matter of **how everything is in every respect**. Thus we are going beyond Lewis in making a proposal about which subject matter **m** deserves to be considered "the subject matter of S."

Our intuitive notion of "what a sentence is about" is deeply tied up with our notion of the reasons for that sentence's truth-value. A sentence that is about ravens rather than writing desks is true (or not) because of what ravens are like and not because of what writing desks are like. I propose that subject matter is that which changes when the reason(s) why a sentence is true change; it is a partition of logical space such that worlds are cellmates iff S is true for the same reason in them.

If we give A's truth-conditions by saying of each world *whether* A is true in that world, we give its subject matter by saying *why* it is true in that world. To specify S's subject matter is just to enumerate the possible reasons for S to be true, in other words to enumerate S's truthmakers. One says what S is about by giving the various reasons it might be true. In some contexts it will be helpful to look also at the various reasons S might be false. This, in an inspired turn of phrase suggested by John MacFarlane, can be considered, not S's subject matter, but rather its *subject anti-matter*. The two together are S's *overall subject matter.* [3]

That's our theory of sentential subject matter; it's the set of truthmakers, or in the case of subject anti-matters, the set of falsity-makers. The next step is to put THIS together with the explanation above of subject-matter inclusion to get a story of what it means for one sentence's subject matter to include another sentence's subject matter (and likewise for subject anti-matters). What you get, which should seem more or less plausible even sparing you the gory details, is

> (4) B's subject matter is part of A's iff each of B's truthmakers is implied by one of A's truthmakers. B's subject anti-matter is part of A's iff each of B's falsemakers is implied by one of A's falsemakers

So, the subject matter of *Deb is big and strong* includes that of *Deb is big* because every truthmaker for the latter is implied by one for the former. The subject matter of *Deb is big* does not include that of *Deb is big and strong* because not every truthmaker for the latter is implied by one for the former. That Deb is strong is not implied by any truthmaker for *Deb is big.*

5 Content-parts and partial truth

Now we are in a position to define content-part.

> (5) B is *part* of A iff the inference from A to B is both truth-preserving—A implies B—and aboutness-preserving—A's subject matter includes B's subject matter and likewise for subject anti-matters.

[3]So, S and its negation have different subject matters *strictly speaking*, but the same *overall* subject matter, since S's subject matter is ¬S's subject anti-matter and vice versa.

If we use $\mathbf{S}^{\mathrm{t}}$ as a variable ranging over S's truth-makers and $\mathbf{S}^{\mathrm{f}}$ as a variable ranging over S's falsity-makers, we can put it like this:

(6) B is *part* of A iff

1. A implies B
2. each $\mathbf{B}^{\mathrm{t}}$ is implied by an $\mathbf{A}^{\mathrm{t}}$
3. each $\mathbf{B}^{\mathrm{f}}$ is implied by an $\mathbf{A}^{\mathrm{f}}$ [4]

That completes the definition of content-part.[5] It tells us, given two hypotheses A and B, whether B is part of A. So for instance *Pam is healthy* is part of *Pam is healthy and weaithy* because any truthmaker for *Pam is healthy* is implied by one for *Pam is healthy and wealthy*, and every falsemaker for *Pam is healthy* is implied by, is indeed identical to, a falsemaker for *Pam is healthy.* Compare this with a different case: *Pam is healthy* vs *Pam is healthy or wealthy.* This time there is no inclusion because some truthmakers for *Pam is healthy or wealthy* are not implied by any truthmaker for *Pam is healthy.* Similar reasoning shows that *Everyone is healthy* includes *Pam is healthy*, but *Pam is healthy* does not include *Someone is healthy.*

Now we know how to *recognize* content-parts when we see them; A's parts are those its implications whose truthmakers (falsemakers) are implied by truthmakers (falsemakers) for A. However it would also be good to know how to *make* content-parts. Why? Our original problem was to say what it is for A to be true about a particular subject matter. I assume that

(7) A is true about **m** iff the part of A that concerns **m** is true simpliciter.

But now what is "the part of A that concerns **m**"? It will have to be constructed somehow out of A and **m**.

An example to fix ideas. Suppose that we have a strange reading of Genesis which has God tripling the number of stars every day, the process beginning on the third day after creation. The clearest way to express the hypothesis is *The number of stars on day n is* $3^{(n-3)}$.

Our reading of Genesis also tells us that God never created numbers, though, so we don't believe in numbers. Really, we think, our hypothesis only partly true, true in what it says about the concrete world. How do we home in on the part of our hypothesis that concerns stars and the like as opposed to numbers?

How we approach this depends on whether we take ourselves to be looking for a sentence or something non-linguistic that a sentence if there were one would express. I take it we are looking more for the latter. Falstaff has probably not taken care to keep his statements about Jones separate from his statements about Smith et al; he was talking about them all together and there's no guarantee that a sentence can be found that sums up what he said just about Jones. Likewise there is no guarantee that a sentence can be found that sums up what *The number of stars on day* $n = 3^{(n-3)}$ says just about the stars.

What we are looking for, then, is a proposition that might be expressible in English or might not. This proposition will have to include a truth-conditional content **B**, certainly. But it will have to include a subject matter **b**, too—for the sought after proposition is to be *part* of A and parthood

[4] In practice this means that every $\mathbf{B}^{\mathrm{f}}$ *is* an $\mathbf{A}^{\mathrm{f}}$. This is because B's falsemakers *already* imply that A is false, given that A implies B. For A's falsemakers to add in unneeded extra detail would seem prima facie to be a violation of proportionality.

[5] Compare the proposals in [?] and [?].

requires subject matter inclusion. So: what A says about **m** will be a truth-conditional content **B** with associated subject matter **b**: a lewisian set-of-worlds intension taken together with a number of subsets thereof which explain (in some world or other) why the set-of-worlds intension is true or false. I will refer to this kind of amalgam as a *directed proposition* and write it in bold italics. The question is, given

> One, a sentence A with directed propositional content ***A*** ,

and

> Two, a lewisian subject matter **m** (a partition of logical space),

can we find

> Three, a directed proposition ***B*** (= ⟨**B**, **b**⟩) such that a sentence *if there were one* that expressed ***B*** would count as the part of A that concerned **m**.[6]

The answer is that we can. Let me not go into details here but it is pretty clear how to do it. You form ***B*** by making ***A*** blind to distinctions between **m**-equivalent worlds. **B** in other words is a proposition true (false) in a world iff that world can (can't) be brought into compliance with A while leaving its **m**-condition unchanged.

So, the part of A = *The number of stars on day n =* $3^{(n-3)}$ that concerns the concrete world is true in <u>w</u> iff <u>w</u>'s concrete condition puts no obstacles in the way of A's truth there, witness that A is true in a world concretely indistiscernible from <u>w</u>. The reasons for its truth in a world are the reasons for A's truth, watered down so as not to distinguish concretely equivalent worlds—so, the fact that there is one star on the third day, there are three stars on the fourth day, there are nine stars on the fifth day, and so on. It is false in a world for the same reasons that A is false in a world, leaving aside reasons that care about more than how matters stand concretely—the fact that there are a hundred stars on the fifth day, for instance, as opposed to the fact that there fail to be any numbers.

That completes the construction. It shows that truth about a subject matter can indeed be understood as truth of a part—the part that concerns that subject matter. At least, it shows that if **B** as just defined really is part of A. But this follows immediately from the definitions. **B** is true in all A-worlds and their **m**-equivalents. So every A-world is a **B**-world, whence **B** is implied by A. Every truthmaker for **B** is implied by one for A simply because **B**'s truthmakers were constructed by weakening A's truthmakers. Every falsemaker for **B** is implied by one for A simply because **B**'s falsemakers were obtained by selecting from among A's falsemakers the ones that do not distinguish **m**-equivalent worlds.

6 The paradox re-solved

How can it be clearly, indisputably, correct that *The number of dragons is 0*, yet controversial whether *There are such things as numbers*?

Our initial answer was something like this (I embellish a little). *The number of dragons is 0* is *not* clearly correct, if that means clearly correct in its entire truth-conditional content. It is clearly

[6] And which itself to that extent can be considered the part of A that concerns **m**.

correct only relative to the presupposition π that there are such things as numbers. *The number of dragons is 0* becomes just as controversial as π if π is not presupposed. But π IS presupposed by *The number of dragons is 0.* It is presupposed semantically for Strawsonian reasons: a definite description *the so and so* triggers the presupposition that there is a so and so. It is presupposed pragmatically because to ask whether *The number of dragons is 0* is to ask a question raised by that sentence *in particular*—a question not equally raised by the common run of other applied mathematical sentences, e.g., *The number of Martian moons is 2.* The question of whether π holds is equally raised by *The number of Martian moons is 2*; so it is not what the speaker was wondering about.[7]

So: *The number of dragons is 0* is clear only relative to the presupposition that *There are such things as numbers*, a presupposition that is not easily imagined away. What about *There are such things as numbers* itself? It too is clear relative to the presupposition that *There are such things as numbers.* But just as it is hard to hear *The number of dragons is 0* as free of the presupposition that π, it is hard to hear *There are such things as numbers* as presupposing that π—that would be to hear π as presupposing itself. *There are such things as numbers* strikes us as unobvious because the presupposition that protected its predecessor from philosophical scrutiny is no longer in place.

Let me now put this solution in a large context using notions developed in the last few sections.

First, when a sentence S presupposes (rather than asserts) something π, truth-value judgments about S are as far as possible insensitive to π; they are driven by the truth or falsity of what S *adds* to π.

Second, this holds in particular when S = *The number of dragons is 0* and $\pi = \omega =$ *There are such things as numbers.* Truth-value judgments about *The number of dragons is 0* are driven by the truth or falsity of what that statement adds to the existence of numbers.

Third, what S adds to the existence of numbers is what S says about the number-free part of reality—henceforth the *concrete* part.

Fourth, what *The number of dragons is 0* says about the concrete part of reality is that *There aren't any dragons.* That there aren't any dragons is clearly true. S strikes us as clearly true because it is clearly right about the subject matter under discussion.

Fifth, however, although S presupposes π, its consequence $\omega =$ *There are numbers* does not inherit the presupposition. For π in this case is *identical* to ω; and ω does not presuppose itself. Truth-value judgments about *There are numbers* accordingly do *not* reflect the truth of what it says about the number-free part of reality (that would be silly). They reflect the truth-value of *There are numbers* itself. And *There are numbers* itself is hotly debated. It strikes us as controversial because it *is* controversial.

To say it all in one breath: the reason S strikes us as clearly correct is that in evaluating S we focus on the part of S that ignores the numbers; but when we get to evaluating *There are numbers* there is no comparable part to evaluate and we are driven back to the (controversial) whole.

This is not yet an account of how insiders (π-presupposers) are able to make themselves understood to outsiders, but it suggests such an account. Insiders, when they utter the sentence S, are asserting the part of S that concerns how matters stand *numberss apart.* That part of S does not presuppose π, so outsiders can understand it and potentially agree with it. What outsiders understand and potentially agree with is that S is true insofar as it concerns the (number-free) subject matter under discussion.

[7]The point more generally is this: any discourse presumes a distinction between correct statements and those that are incorrect; this distinction is obliterated if statements are counted incorrect on the basis of a presupposition they all share.

7 Digression: Pure math

When Carnap states the paradox, he uses not an applied mathematical sentence (like *The number of dragons = 0*) but a pure-mathematical sentence like *There are primes over 10, all of which are odd.* How can it be clear that there are prime numbers over 10, all of them odd, when the existence of numbers as such is controversial?

The full content of S = *There are primes over 10, all of which are odd* is controversial, according to the line I've been running, since its truth depends on the existence of numbers. If it strikes us otherwise, that is because the statement has an uncontroversial part—a part that is interestingly true and remains so even if numbers do not exist. This time, though, it is harder to see what the uncontroversially true part might be. Wouldn't it follow from the non-existence of numbers that, as Hartry Field once suggested, existential numerical claims were all automatically false, and universal numerical claims were all automatically true? That would seem to leave little room for false numerical claims with interestingly true parts.

But the idea that *Primes over 10 are even* is, in the absence of numbers, every bit as true as *Primes over 10 are odd* is not at all obvious. It assumes that we are dealing with *enumerative* generalizations about *whatever numbers there happen to be.* And I don't know why we would assume this, any more than we would assume *Objects with no net forces on them accelerate* was an enumerative generalization about whatever objects there happen to be. *Objects with no forces on them accelerate* sounds false, even if there are no objects like that, simply because accelerating is *physically unlawful behavior* for them. It's exactly the same with *Primes over 10 are even*; it sounds false whether there are primes over 10 or not, simply because this is mathematically unlawful behavior for primes that big.

How are we to explain the intuitive falsity of uninstantiated generalizations like those just discussed? The obvious explanation is that they have two parts, one saying how objects of the relevant sort behave *qua objects of that sort*, the other (perhaps presupposed rather than asserted) to the effect that objects of the relevant sort exist. This applies not just to universal claims about numbers but also existential ones. *There are infinitely many primes* has as its first part *Numbers are of a type to include among them infinitely many primes* and as its second part *…and the type is instantiated—numbers exist.* When we say *There are infinitely many primes*, we are putting the first part forward as true but not the second. Alternatively we are putting the whole statement forward as true-about-a-certain-subject-matter, that subject matter being the sosein of numbers rather than their sein; this comes to roughly the same given our theory of content-parts.

Back to Carnap's paradox in its pure mathematical version. We know very well that there are prime numbers over 10, but it is highly debatable whether there are numbers. How can that be? Surely if we know a stronger statement, we know its weaker implication? Well, but maybe what we know in the first case is not the full statement, but the part that concerns numbers' sosein: *Numbers are of a type to include infinitely many primes. There are numbers* is different in that the sosein part is transparently silly: *Numbers are of a type to include numbers.* That can't be what we are putting forward as true, or claiming to know. So we are thrown back on the other part: there are numbers. And whether there are numbers is not something we can claim to know just on the basis of mathematical competence.

8 Jamesian apologetics

Why bother with partial truth as we have defined it? Why utter false sentences with true bits rather than just the true bits? The answer was given a long time ago by William James (1979, 31-32):

> a rule of thinking which would absolutely prevent me from acknowledging certain kinds of truth if those kinds of truth were really there, would be an irrational rule.

This is usually read as a plea for epistemic boldness; if "acknowledging certain truths" carries a risk of acknowledging the odd falsehood, well, that's a price worth paying. But one can also hear it as a plea for semantic boldness. Suppose that certain truths are best or only accessed as scattered parts of larger falsehoods (or larger hypotheses that might be false). (Quine quote about convenient myths containing the truth as a scattered parts) Dallying with the larger falsehoods may be a price worth paying. The difference with James is that it is not the falsity of one statement tolerated for the sake of another's truth, but the falsity of one statement tolerated for the sake of the truth, or anyway partial truth, of the very same statement — for the sake of the truth this statement contains.

Of course, this apology for partial truth is only as good as the premise that certain truths are only or best accessed as part of larger falsehoods. One can certainly see how the premise *might* be true. Look at the construction of the true part a moment ago; the construction yielded a true meaning but not a true sentence whose meaning it was. The only sentence available is the original one which we're supposing is or may be false. One can describe the intended meaning and endorse it but there is no obvious way to assert it. What option do we have then but to put the possibly untrue sentence forward in a quasi-assertional spirit? Our plea to the charge of speaking falsely will have to be "guilty with an excuse": part of what we said was true, we don't know how to assert just that part, and we did our best to clue you in to which part it was; it's the part about such and such a subject matter.

So we can see how there might be a Jamesian justification for speaking more than the truth, or more than what we know to be true. It would be nice to have some actual examples though. I am not sure I have any actual examples but here are some possible examples.

Applied math Suppose I am not a platonist. I do not believe in numbers and functions. Like anyone else I want to use mathematical physics to describe the physical world; I want to be able to say, for instance, that a projectile's escape velocity is such and such a function of how much it weighs, the take-off angle, and so on. The escape velocity formula is not true in my view, just as Falstaff's testimony was not true. But it is true in what it says about the physical world. I don't know how to express what it says about the physical world otherwise than to say: the part that concerns the physical world. It is true that my construction of that part runs essentially through platonist worlds.

Am I committed then to saying numbers are metaphysically contingent? No. A world for these purposes is just a coherent scenario. At most I am committed to platonism being conceptually possible. And that's a commitment I happily own up to. Platonism may be untrue, but I never said it was conceptually incoherent! So there's my Jamesian justification.

Negative singular existentials I believe that, as I'm tempted to put it, Pegasus fails to exist. But I am not a Meinongian: Pegasus doesn't subsist either; it has no kind of being whatsoever; "Pegasus" is an entirely empty name. This puts me in a bit of a bind, since sentences with empty names in them are false, or anyway untrue. If Pegasus fails to exist is in my view untrue, why do I say it?

Well, it is partly true: it is true about the things that exist. To go by our definitions above, its truth about the things that exist consists in the full truth of that part of Pegasus doesn't exist that concerns the existing things. This is obtained in just the way one obtains the part of *The # of planets = 8* that concerns the concrete things. Recall that the part of *The # of planets = 8* that concerns the concrete things is a proposition true in all worlds concretely equivalent to a Platonic world in which the # of planets really is 8. Likewise the part of Pegasus doesn't exist that concerns the existing things is a proposition true in all worlds with the same existing things as a Meinongian world where Pegasus really does not exist. That part is true because although it is not true in our world that Pegasus is nonexistent, it is true in a world otherwise like ours but with Pegasus tacked on as a subsisting thing.

The upshot is that when I say Pegasus fails to exist, the part of that that I care about, the part that concerns the totality of what exists, is perfectly true. And since I know no other way to put that part forward than to say Pegasus fails to exist, that's what I say. It is true that my construction of the true part runs essentially through Meinong worlds, worlds where Pegasus subsists. But that's OK for the same reason running through a Platonic world is OK. Meinong may have been wrong, but the view is not conceptually incoherent! There's my Jamesian justification.

Standards of precision I believe that, as I am tempted to put it, I am 5 feet 9 inches tall. This puts me in a bit of a bind because I think the statement I am 5 feet 9 inches tall is false. For I am less than 5 ft 9 inches tall (closer to 5 ft 8) and you can't be 5 ft 9 tall and less than 5 ft 9 at the same time. I believe that, as I'm tempted to put it, there were a million men in attendance at the Million Man March. But I also think that "a million" means exactly a million— it entails, for instance, "not ten less than a million". So my statement is almost certainly false. How then can I make the statement?

Well, my subject matter when I said I was 5 foot 9 was how tall people are in inches, in the sense of which height in inches their precise height is closest to. My statement was false, but true about the subject matter of height in inches. My subject matter when I said there were a million men in attendance was attendance in hundreds of thousands, or perhaps lower-bounds-on-attendance-in-hundreds-of-thousands. To express the true part directly, I would have to say that 5'9" is the height in inches closest to my precise height. I could, I suppose, but this way of putting it is ugly and inconvenient and require explicitness about something that I may find it difficult to state explicitly. Better to stick with the original precise statement and let the standards of precision rise and fall with the subject matter.

Pure math[8] I am a non-platonist, let's say this time a nominalist. I think it is false that *There are primes over 10, none of which are even.* Like anyone else, though, I want to be able to say it. Why? Well, if I'm to stick with the program, it is because the statement has a part that I do believe, a part that is interestingly true in my view, and remains so even if numbers do not exist. The problem is, this time it is harder to see what the true part might be. Doesn't it follow from the denial of numbers that, as Hartry Field once suggested, existential numerical claims are one and all false, and universal numerical claims are one and all true? That would seem to leave little room for interestingly true parts to false numerical claims.

But the idea that *Primes over 10 are even* is every bit as true as *Primes over 10 are odd* assumes that we are dealing with enumerative generalizations about whatever numbers there happen to be. And I don't know why we would assume this, any more than we would assume *Objects with no*

[8]The next few paragraphs repeat material from the previous section.

net forces on them accelerate to the speed of light is an enumerative generalization about whatever objects there happen to be. *Objects with no forces on them accelerate* sounds false, even if there are no objects like that, simply because accelerating is physically unlawful behavior for such objects. It is exactly the same with *Primes over 10 are even*; it sounds false whether there are primes over 10 or not, simply because this is mathematically unlawful behavior for primes other than 2.

How are we to explain the intuitive falsity of uninstantiated generalizations like these? The obvious explanation is that they have two parts, one a law saying how objects of the relevant sort behave qua objects of that sort, the other (perhaps presupposed rather than asserted) to the effect that objects of the relevant sort exist. There are infinitely many primes has as its first part *Numbers are of a type to include infinitely many primes*, and as its second part *...and there are some.* Nominalists when they say *There are infinitely many primes* are putting the first part forward as true but not the second. Alternatively we could say they are putting the whole statement forward as true-about-a-certain-subject-matter, that subject matter being the sosein of numbers rather than their sein.

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