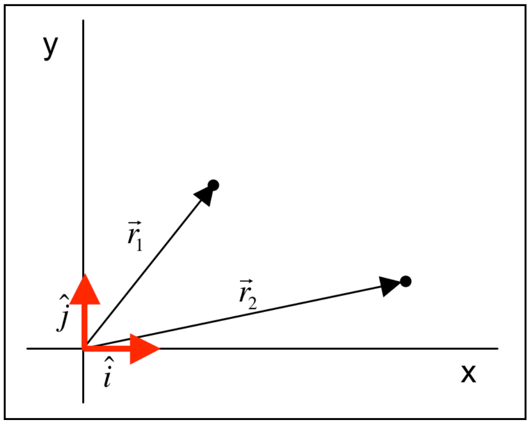
Displacement and Velocity in 2-D



Rectangular coordinates:

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$$

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$$

Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

Average velocity = displacement / time $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{x,avg}\hat{i} + v_{y,avg}\hat{j}$

Instantaneous velocity: $\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$

Average acceleration: $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = a_{x,avg}\hat{i} + a_{y,avg}\hat{j}$

Instantaneous acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} = a_x\hat{i} + a_y\hat{j}$

Example:

The position of an electron is given by: $\vec{r}(t) = \left[(3.00\,{}^{m}\!/\!{}_{s})t\right]\hat{i} + \left[(-4.00\,{}^{m}\!/\!{}_{s^2})t^2\right]\hat{j}$.

a. What is $\vec{v}(t)$?

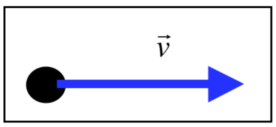
b. At t = 2.00 s, what is $\vec{v}$?

c. What is $\vec{a}(t)$?

Last time – one form of 2-d motion we looked at was projectile motion (where there was a constant force in one direction – down, due to gravity)

Now we will look at another form of 2-d motion:

Uniform Circular Motion – particle travels around a circular path or arc at a **constant speed**.



How can you change the direction of this particle's motion without changing its speed?

➔ Apply a force perpendicular to the velocity

Demonstration – mass on a string

Tension: force that changes the direction of motion
always points toward the center of the circular path
"centripetal" – center-seeking

A change in the direction of a velocity vector ➔ acceleration (centripetal)

Example: A car is traveling around a circular curve so that at one instant the car is moving at 30 m/s North and 10 s later the car is moving at 30 m/s East. What is the acceleration of the car?

$\vec{v}_1 = (30\,{}^{m}\!/\!_{s})\hat{j}$ and $\vec{v}_2 = (30\,{}^{m}\!/\!_{s})\hat{i}$

$\Delta t = 10s$

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\vec{v}_2}{\Delta t} - \frac{\vec{v}_1}{\Delta t} = \frac{(30\,{}^{m}\!/\!_{s})\hat{i}}{10s} - \frac{(30\,{}^{m}\!/\!_{s})\hat{j}}{10s} \neq 0$ Why is the acceleration $\neq 0$?

➔ Because the velocity **changes** direction.

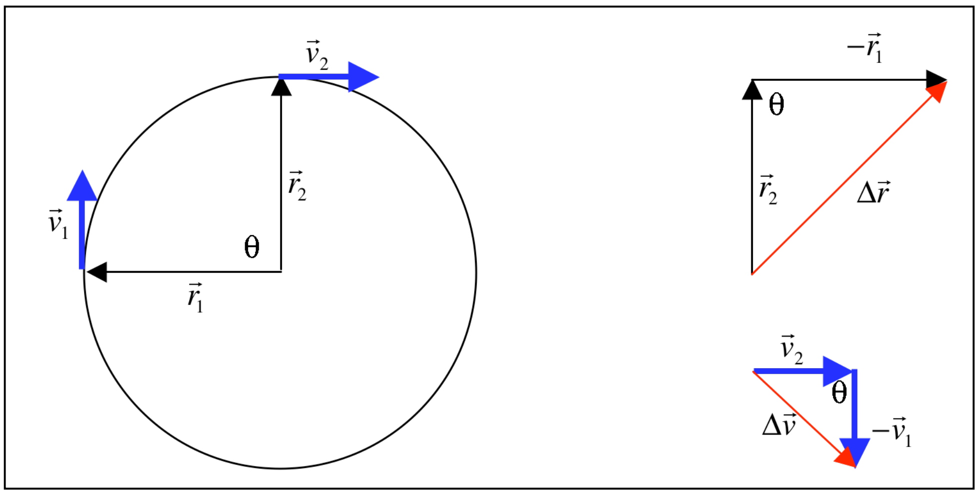
(Example, continued)

$$\vec{a} = \left(3\,{}^{m}\!/_{s^2}\right)\hat{i} - \left(3\,{}^{m}\!/_{s^2}\right)\hat{j} = \left(3\,{}^{m}\!/_{s^2}\right)\hat{i} + \left(-3\,{}^{m}\!/_{s^2}\right)\hat{j}$$

$$|\vec{a}| = \sqrt{\left(3\,{}^{m}\!/_{s^2}\right)^2 + \left(-3\,{}^{m}\!/_{s^2}\right)^2} = 4.2\,{}^{m}\!/_{s^2}$$

$$\theta = \tan^{-1}\frac{\left(-3\,{}^{m}\!/_{s^2}\right)}{3\,{}^{m}\!/_{s^2}} = \tan^{-1}(-1) = -45°$$

Let's show this graphically:



These are similar triangles – since $\vec{v} \perp \vec{r}$, θ will always be the same angle in both triangles.

Let v be the speed and R be the radius of the circular path.

Ratio of the magnitudes: $\dfrac{\Delta v}{v} = \dfrac{\Delta r}{R}$ ➔ $\Delta v = \dfrac{(\Delta r)(v)}{R}$

Acceleration: $a = \dfrac{\Delta v}{\Delta t} = \dfrac{(\Delta r)(v)}{(\Delta t)(R)} = \dfrac{v^2}{R}$

➔➔➔ $\boxed{a_c = \dfrac{v^2}{R}}$ ➔ centripetal acceleration (magnitude, direction is always toward the center of the circular path)

1. In uniform circular motion, is the velocity constant? Is the acceleration constant? Explain.

2. In uniform circular motion, the net force is perpendicular to the velocity and changes the direction of the velocity but does not change the speed. If a projectile is launched horizontally, the net force (ignoring air resistance) is perpendicular to the initial velocity, and yet the projectile gains speed as it falls. What is the difference between the two situations?

➔ In uniform circular motion, the acceleration always points toward the center of the circular path – perpendicular to the velocity the whole time. In projectile motion, the acceleration always points straight down – it is perpendicular to the velocity for only an instant (at the max height).

An Earth Satellite

Circular orbit, altitude = 640 km above Earth's surface.

Period = 98.0 min (time for a particle to go around a closed path once)

What is the satellite's speed and centripetal acceleration?

Keeping Mars in Orbit. Although the planet Mars orbits the Sun in a Kepler ellipse with an eccentricity of 0.09, we can approximate its orbit by a circle. If you have faith in Newton's laws then you must conclude that there is an invisible centripetal force holding Mars in orbit. The data on the orbit of Mars about the sun are shown below:

m_{sun} = 2.00 x 10^{30} kg

m_{Mars} = 0.107 Earth masses or 6.42 x 10^{23} kg

d_{Mars} = 2.28 x 10^{11} m = distance from the sun (= radius of circular orbit)

<v> = 24.13 km/s (mean orbital speed)

a. Centripetal force needed to hold Mars in orbit?
b. Direction of force?
c. Most likely source of this force?
d. Anything in common with the force that attracts objects to Earth?

3. Explain why the force of gravity due to the Earth does not pull the Moon in closer and closer on an inward spiral until it crashes into the Earth?

Find the tension in the string

Calculate the period, measure the radius of the path, calculate the speed.