Strategic Complementarities and Search Market Equilibrium[†]

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In this paper, we apply supermodular game theory to the equilibrium search literature with sequential search. We identify necessary and sufficient conditions for the pricing game to exhibit strategic complementarities and prove existence of equilibrium. We then show that price dispersion is inherently incompatible with strategic complementarities in the sense that the Diamond Paradox obtains when firms are identical and is robust within the class of search cost densities that are small near zero and support strategic complementarities. We also show that a major criticism of the literature, that agents act as if they know the distribution of prices, can be justified in the sense of convergent best response dynamics.

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1. Introduction

In this paper, we apply concepts and tools from supermodular game theory to address three fundamental and long-standing open questions in the equilibrium search literature. The first question is: what is the nature and source of observed price dispersion, which has been extensively documented in real-world markets? At present, the search literature contains several distinct classes of models that can explain this phenomenon, including sequential search models such as Reinganum (1979), Rob (1985), and Stahl (1989, 1996); nonsequential search models such as Burdett and Judd (1983) and Janssen and Moraga-González (2004); and information clearinghouse models such as Baye and Morgan (2001). For a survey of this literature, see Baye, Morgan, and Scholten (2006).

In this paper, we consider the class of equilibrium search models with sequential search developed by Carlson and McAfee (1983) and Bénabou (1993), which allows for potential bilateral heterogeneity in consumers' search costs as well as firms' production costs. According to conventional wisdom, distilled from Rob's (1985) seminal contribution, dispersion can be a purely informational phenomenon in this class of models in the sense that it can be generated solely by heterogeneous search costs, without any heterogeneity in firms' production costs. An important aspect of his model, however, is that demand is assumed to be perfectly inelastic at one unit with infinite willingness to pay, which cannot be literally true with finite income. This assumption rules out *a priori* the Diamond (1971) equilibrium where identical firms almost all charge the common monopoly price, because the latter is not well-defined. As a result, dispersion occurs in Rob's model under quite general conditions.

In contrast, we show that when demand is potentially downward-sloping with finite willingness to pay, a Rob-type condition applies (the value of the density of search costs at zero is sufficiently small), and the pricing game exhibits strategic complementarities, the unique equilibrium with identical firms (same production cost) is the Diamond one. In that case, no amount of heterogeneity in search costs can overcome the Diamond Paradox as long as the distribution of search costs remains within the class that supports strategic complementarities. Under these conditions, it seems that price dispersion either requires an atom in the distribution of search costs at zero as in Stahl (1989, 1996) or some additional heterogeneity like heterogeneous production costs as in Reinganum (1979) or heterogeneous sampling probabilities as in Hortaçsu and Syverson (2004). Indeed, we show that a simple Reinganum-type condition on firms' costs ensures that all equilibria are dispersed. In that case, however, dispersion is not a purely informational phenomenon because it is driven at least in part by *technological* conditions.

Since its inception in the early 1960s, a major recurring criticism of the literature has been that consumers and firms act "as if" they know the distribution of prices. E.g.,

> These results depend on the assumption that the searcher behaves as if he knows the distribution of prices. In any economic context, this is a very bad assumption. Little is known about the nature of price distributions, and it seems absurd to suppose that consumers know them with any degree of accuracy.
>
> Rothschild (1974)

> Perhaps the most restrictive and least palatable assumption of the elementary search model is the supposition that the offer distribution F is known.
>
> Lippman and McCall (1981).

Although this criticism is valid for any static game-theoretic equilibrium concept, it applies with particular force to this literature, whose explicit aim after all is to characterize markets with informational imperfections. Moreover, it seems clear that real-world consumers (and firms) do not have anything close to the required information.

To address this issue, Bénabou and Gertner (1993) and Dana (1994) construct search models where firms have private information about their costs and consumers learn about the distribution of prices while they search, given their knowledge of the distribution of production costs as well as firms' pricing strategies. Although this approach generates important insights (e.g., about the link between dispersion and unanticipated inflation), these informational requirements are clearly very strong. Another approach is that of Rauh (1997), where agents only know finitely many moments of the distribution of prices. Although this is weaker than knowing the entire distribution (all infinitely many moments), it is still not clear how agents obtain this information, which is contemporaneous.

In this paper, we pursue a dynamic approach to this issue and show that "as if" knowledge of last period's distribution of prices can be sufficient in the sense of convergent best response dynamics, where consumers' reservation levels and firms' profit-maximizing prices are best responses against the distribution of prices in the previous period. In contrast, Hopkins and Seymour (2002) show that the mixed-strategy equilibria in Varian (1980) and Burdett and Judd (1983) are generally dynamically unstable with respect to a broad class of learning rules that includes fictitious play, which is analogous to best response dynamics. Although the aforementioned models can generate dispersion with little or no *ex ante* heterogeneity, their mixed-strategy equilibria tend to be unstable, whereas the model in this paper can possess stable dispersed equilibria, but in general dispersion requires heterogeneity in both search and production costs.

The final issue concerns existence of search market equilibrium. In this literature, the pricing game consists of a continuum of firms whose profits depend on their own prices as well as the *distribution* of prices in the market. Unfortunately, little seems to be known about games with this structure, so existence is a nontrivial problem and the main methodological innovation of the present paper is its appeal to supermodular game theory, as developed by Milgrom and Roberts (1990), Topkis (1979, 1998), and Vives (1990, 1999). Indeed, strategic complementarities seem quite natural in the current context involving price competition with a homogeneous good: if firm i maintains its price while all of its competitors raise theirs, then i should receive more visits from potential customers, allowing it to raise price as well.

We therefore seek conditions on the density of consumers' search costs that support strategic complementarities. When that density is decreasing, as in Rob (1985) and the empirical work of Hong and Shum (2006), we show that a simple sufficient condition is a hazard condition which requires the density to be relatively high and flat, like the uniform distribution. In the case of a hill or bell-shaped density, like the truncated normal, we show that the pricing game is strictly supermodular if a similar hazard condition holds and the proportion of inactive searchers (those who search exactly once with probability one) is at least 50% of the consumer population. As discussed more fully below, the latter assumption is consistent with the structural estimates of Hong and Shum (2006), De los Santos (2007) (who observes consumers' search behavior directly), and Wildenbeest (2007), but higher than the estimates in Hortaçsu and Syverson (2004), Moraga-González and Wildenbeest (2008), and Moraga-González, Sándor, and Wildenbeest (2007). It is also consistent with the following assessment of the relevant marketing literature:

> External information search is skewed toward limited search, with the greatest proportion of consumers performing little external search immediately prior to purchase. Surveys of shopping behavior have shown a significant percent of all durable purchases are made after visiting only one store... Measures of the use of personal and nonmarket sources also show somewhat limited levels of search... Based on six separate studies that span more than 30 years, two product categories, four services, and two countries... approximately half of the purchases are preceded by virtually no external information search; about one-third are associated with limited information search; and only 12 percent involve extensive information seeking prior to purchase.
>
> Hawkins, Best, and Coney (1995, p. 451)

Although games with a continuum of players whose payoffs depend on the distribution of actions have not been analyzed in the literature on supermodular games, we show that several of the main results extend to search models. In particular, we show that when the pricing game exhibits strategic complementarities there exists a largest (high-price) and smallest (low-price) pure-strategy equilibrium, as in standard supermodular games. Moreover, existence is more than a theoretical curiosity, since we use it to prove our results on price dispersion: when firms are identical, the Rob-type condition mentioned earlier implies a unique symmetric (single-price) equilibrium and the aforementioned extremal equilibria are symmetric. It follows that the largest and smallest equilibria are symmetric and the same, so the Diamond equilibrium is unique. A basic insight of the paper, therefore, is that price dispersion is inherently incompatible with strategic complementarities, where strategic variables tend to be set at comparable levels, whereas dispersion requires that some firms set high prices and others low ones.

If the pricing game exhibits strict strategic complementarities, we can also apply standard results from supermodular game theory on the stability of the best response dynamics, suitably adapted to the present context. Specifically, from any initial condition the best response dynamics approach the interval of strategy profiles determined by the largest and smallest equilibria, so a unique equilibrium is globally stable. An important special case is when the Diamond Paradox obtains and the unique Diamond equilibrium is globally stable as originally discovered by Diamond (1971). The literature on supermodular games is therefore utilized in a crucial way in proving all three main results of the paper on existence, price dispersion, and dynamics.

The plan for the rest of the paper is as follows. In section 2, we develop the model and characterize some of its essential features. In section 3, we derive conditions such that the pricing game exhibits strategic complementarities and establish the existence of search market equilibrium. We study price dispersion in section 4 and the stability of the best response dynamics in section 5. Section 6 concludes. All proofs not in the text are in the appendix.

2. The Model

The model is similar to those in Reinganum (1979), Carlson and McAfee (1983), Rob (1985), Bénabou (1993), and Rauh (2007). Let $I = [0,1]$ be the set of firms and $[0,\theta]$ the set of consumers, so the buyer-seller ratio is θ. Let $m(i)$ be the constant marginal cost of production of firm i, $\underline{c} = \inf_i m(i)$, and $\overline{c} = \sup_i m(i)$.[1] Except for their costs, firms are otherwise identical. We assume all consumers have the same indirect utility function $v(p)+y$, where p is price and y is income. Let $\overline{p}$ be the maximum price consumers are willing to pay for the good, where $\overline{c} < \overline{p} < \infty$. Assume v is twice continuously differentiable on $[0,\overline{p}\,]$ (one-sided derivatives at the endpoints). The demand curve is therefore $x(p) = -v'(p)$, which is continuous on $[0,\overline{p}\,]$. The following assumptions are general enough to cover most of the cases considered in the literature.[2]

Assumptions 1. *(i)* $x > 0$ *on* $[0,\overline{p})$ *and* $x = 0$ *on* $(\overline{p},\infty)$. *(ii)* x *is (at least weakly) decreasing on* $[0,\overline{p}\,]$.[3] *(iii) For all* $c \in [\,\underline{c},\overline{c}\,]$, *the monopoly problem*

$$\max_{p\in[c,\overline{p}\,]} \pi^M(p,c) = (p-c)x(p) \tag{1}$$

has a unique solution $p_M(c) \in (c,\overline{p}\,]$ *and* $\pi^M_p \equiv \partial\pi^M/\partial p > 0$ *on* $[c, p_M(c))$.

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[1] We assume all exogenously given functions such as $m : I \to [\,\underline{c},\overline{c}\,]$ are at least Borel measurable.

[2] E.g., it is often assumed that demand is perfectly inelastic at one unit up to $\overline{p}$, so $x = 1$ on $[0,\overline{p}\,]$. The above assumptions also hold when demand is downward-sloping, twice differentiable on $[0,\overline{p}\,]$, and not too convex, so that π^M is strictly concave. An example is linear demand $x(p) = a - bp$, where $a, b > 0$ and $\overline{p} = a/b$. Note that we make no assumption about the value of $x(\overline{p})$ since $x(\overline{p}) = 1$ in the first example but $x(\overline{p}) = 0$ in the linear case. It is easy to show that $p_M(c)$ is increasing in c.

[3] A function f is *increasing* if $x > x'$ implies $f(x) \geq f(x')$ and *strictly increasing* when the latter inequality is strict. We define *decreasing* and *strictly decreasing* similarly.

In this context, a *strategy profile* is a (measurable) function f from the set I of firms to prices $[0, \overline{p}\,]$. Since no firm prices below marginal cost or above its monopoly price, let $P_i = [m(i), p_M(m(i))]$ and $\mathcal{P}$ be the set of all strategy profiles such that $f(i) \in P_i$ for all i.

We assume consumers search sequentially and that the first price quote is free.[4] Let F be the cumulative distribution function (cdf) of prices and r the lowest price the consumer has observed so far. As is well-known [see Bénabou (1993)], the marginal benefit of another search is

$$\Gamma(r, F) = \int_0^r x(p)F(p)\, dp. \tag{2}$$

Proposition 1 below establishes some useful properties of Γ and introduces the parameter $\tilde{s}$, which will play a prominent role throughout the paper. Let $\mathbf{R}$ be the set of real numbers, $\mathbf{R}_+$ the set of nonnegative real numbers, and $\mathcal{D}$ the set of all cdfs with support in $[0, \overline{p}\,]$. We define a partial order $\geq$ on $\mathcal{D}$ as follows: $F \geq F'$ iff $F(p) \leq F'(p)$ for all $p \in [0, \overline{p}\,]$ (i.e., F first-order stochastically dominates F').[5] We write $F > F'$ if $F \geq F'$ and $F \neq F'$.

Proposition 1. *For all $F \in \mathcal{D}$: (i) Γ is absolutely continuous on $[0, \overline{p}\,]$ with derivative $x(p)F(p)$ almost everywhere. (ii) If*

$$z_F = \sup\{p \in \mathbf{R} \mid F(p) = 0\} \tag{3}$$

then $\Gamma = 0$ on the interval $[0, z_F]$, $\Gamma > 0$ on $(z_F, \overline{p}\,]$, and Γ is strictly increasing on $[z_F, \overline{p}\,]$. (iii) $0 \leq \Gamma(r, F) \leq \tilde{s}$ for all $0 \leq r \leq \overline{p}$, where $\tilde{s} \equiv \Gamma(\overline{p}, F_m)$ and F_m is the cdf of marginal costs.

The content of (ii) is illustrated in Figure 1 below, which depicts Γ as a function of r for some fixed F.

Figure 1 Goes Here

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[4] The latter assumption is relaxed in Rauh (2004) and Janssen, Moraga-González, and Wildenbeest (2005). In this paper, we assume the first price quote is free to avoid having to keep track of those consumers who may potentially drop out of the market. As is well-known [see Sargent (1987, Section 2.3)], it makes no difference for our purposes whether search is with or without recall. As Janssen and Parakhonyak (2008) show, however, the optimal sequential search rule is considerably more complex and interesting in the case of costly recall.

[5] Recall that a *partially ordered set* is a set X and a binary relation $\geq$ on X which is reflexive, transitive, and antisymmetric.

In accordance with (ii), Γ is zero between 0 and z_F but is positive and strictly increasing thereafter.

Since $\Gamma(r, F)$ is the marginal benefit of search, a consumer with search cost s stops searching at all prices r such that $\Gamma(r, F) \leq s$, so her reservation level is

$$\sup\{r \in [0, \overline{p}\,] \,|\, \Gamma(r, F) \leq s\}. \tag{4}$$

E.g., in Figure 1 we observe that r_1 is the reservation level of a consumer with search cost s_1. At the extremes, consumers with zero search costs have reservation level z_F and those with $s \geq \Gamma(\overline{p}, F)$ have reservation level $\overline{p}$. Note that a reservation level exists and is unique for each consumer since Γ is continuous and strictly increasing on $[z_F, \overline{p}\,]$.

The significance of the parameter $\tilde{s}$ is that $\Gamma(\overline{p}, F) \leq \tilde{s}$, so consumers with search costs $s \geq \tilde{s}$ have reservation level $\overline{p}$ for all $F \in \mathcal{D}$. These consumers are therefore *inactive searchers* in the sense that they always buy from the first seller they visit, no matter what the distribution of prices. Note that consumers with search costs less than $\tilde{s}$ can also be inactive searchers, but $\tilde{s}$ is convenient because it does not depend on F.

Consumers are identical except for their search costs. Let Q be the cdf of search costs and q the associated probability density function (pdf) with support $[0, \overline{s}\,]$, where $0 < \overline{s} \leq \infty$. Note that $\overline{s}$ can be finite or infinite, so Assumptions 2 below are fairly mild.

Assumptions 2. *(i) $q(s) > 0$ on $(0, \overline{s})$ and zero outside. (ii) q is bounded and continuous on $(0, \overline{s})$.*

Although it creates some (minor) technical issues, we allow q to have discontinuities at $s = 0$ and $s = \overline{s}$ to accommodate the uniform distribution. Recall that pdfs are only defined up to sets of measure zero [Dudley (2002), theorem 5.5.4, p. 175], so we assume $q(0) = 0$ without loss of generality.

The cdf G of consumers' reservation levels is given by

$$G(r|F) = \begin{cases} Q(\Gamma(r, F)) & z_F \leq r < \overline{p} \\ 1 & r \geq \overline{p}, \end{cases} \tag{5}$$

with a potential atom at $\overline{p}$ of size $1 - Q(\Gamma(\overline{p}, F))$. Using Proposition 1, we can represent G on $[0, \overline{p})$ by a density g defined almost everywhere by

$$g(r|F) = G'(r|F) = Q'(\Gamma(r, F))x(r)F(r) = q(\Gamma(r, F))x(r)F(r). \tag{6}$$

All firms face the same demand schedule, informally derived as follows. If firm i charges the price p, its potential customers have reservation levels $r \geq p$. Consumers with reservation level $\overline{p}$ are inactive searchers and are randomly and evenly distributed across all firms, so i expects to sell

$$\theta x(p)[1 - Q(\Gamma(\overline{p}, F))] \tag{7}$$

to them. Let $\theta g(r|F)dr$ be the mass of consumers with reservation level r such that $p \leq r < \overline{p}$. Firm i competes for these consumers with the mass $F(r)$ of firms pricing at or below r, so i expects to sell

$$x(p)\frac{\theta g(r|F)dr}{F(r)} \tag{8}$$

to these consumers. Substituting from (6),

$$\theta x(p)q(\Gamma(r, F))x(r)dr. \tag{9}$$

Summing over all consumers $p \leq r < \overline{p}$ and adding the potential atom at $\overline{p}$, the demand function is $\theta x(p)N(p, F)$, where

$$N(p, F) = \int_{p}^{\overline{p}} q(\Gamma(r, F))\, x(r)\, dr + 1 - Q(\Gamma(\overline{p}, F)) \tag{10}$$

is the "number" of customers. Since θ has no qualitative implications for our results, from now on we set $\theta = 1$ for simplicity.

Proposition 2. *(i) Demand and* N *in (10) are continuous and decreasing on* $[0, \overline{p}\,]$ *for all* $F \in \mathcal{D}$. *(ii)* N *is differentiable on* $[0, \overline{p}\,]$ *except at most two points, corresponding to* $s = 0$ *and* $s = \overline{s}$, *with derivative* $-q(\Gamma(p, F))x(p)$.

We now consider a counterintuitive feature of the model. Suppose all of firm i's competitors raise their prices, while i keeps its price the same at p. Given that the good is homogeneous, one would expect i's sales to increase: $N(p, F) > N(p, F')$ when $F > F'$. As the proof of Proposition 3 shows, however, i's sales could actually *decline* when q is rapidly increasing over some interval.

To see why, consider the truncated normal distribution where $\tilde{s}$ is less than or equal to the mode, so q is increasing on $[0, \tilde{s}]$. When firm i's competitors raise their prices, the first effect is that consumers' reservation levels increase and the atom of consumers with reservation level $\overline{p}$ grows.[6] Since that atom is shared equally among all firms, its growth increases i's sales [see (10)]. The second effect is that the set of consumers with reservation levels less than $\overline{p}$, the ones firms potentially compete for, consists of lower search cost consumers than before. Since q is increasing on $[0, \tilde{s}]$, this represents a shift towards lower search cost consumers who have less mass than before, which reduces i's sales (if q were decreasing on the same interval, the second effect would also increase i's sales). In other words, price increases by a firms' competitors can push the firm towards a smaller niche market with low search cost consumers. The total impact on i's sales depends on the relative size of these two effects. Proposition 3 below provides some simple sufficient conditions that rule out the counterintuitive case.

Proposition 3. *Assume* $\tilde{s} < \overline{s}$ *and* q *is differentiable on* $(0, \tilde{s})$*. A sufficient condition for* $N(p, F) > N(p, F')$ *for all* $p \in P_i$ *and* $F > F'$ *in* $\mathcal{D}$ *is (i)* $q' \leq 0$ *on* $(0, \tilde{s})$ *OR (ii)* $q' \geq 0$ *and* $q'(s) \leq (1/V)q(s)$ *on* $(0, \overline{s})$*, where*

$$V \equiv \int_{\underline{c}}^{\overline{p}} x(p)\, dp. \tag{11}$$

Given the above discussion, these sufficient conditions are intuitive. The assumption $\tilde{s} < \overline{s}$ implies a positive atom of consumers at $\overline{p}$, so the first effect is positive. If q is decreasing on $[0, \tilde{s}]$ then the second effect implies more customers than before, since $F > F'$ shifts the firm's clientele towards low search cost consumers. If, on the other hand, q is increasing on the same interval, then the hazard condition $q'/q \leq 1/V$ puts an upper bound on the slope of q which limits how negative the second effect can be.

Recall that the Rob (1985) search model is the same as that in this paper, except that firms are identical with zero marginal cost and $\overline{p} = \infty$ in his model. In that case, $q' < 0$ is a necessary condition for an absolutely continuous equilibrium cdf of prices to exist [Rob (1985), theorem 4, p. 500]. Indeed, Hong and Shum (2006) used the class of strictly decreasing gamma distributions for their parametric estimates.[7] Other examples covered

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[6] From (2), $F > F'$ implies $\Gamma(r, F) \leq \Gamma(r, F')$ for all $r \in [0, \overline{p}\,]$, so the consumer's reservation level must at least weakly increase.

[7] As noted earlier, there is no contradiction between assuming $q(0) = 0$ and that q is strictly decreasing

by Proposition 3 include the uniform distribution and the truncated normal distribution with sufficiently large σ^2. Indeed, the hazard condition for the latter is

$$\frac{q'(s)}{q(s)} = \frac{\mu - s}{\sigma^2} \leq \frac{\mu}{\sigma^2} \leq \frac{1}{V}. \tag{12}$$

We now define the relevant equilibrium concept. The profits of firm i are

$$\pi(p, m(i), F) = [p - m(i)]x(p)N(p, F) = \pi^M(p, m(i))N(p, F) \tag{13}$$

and its maximization problem is $\max_{p \in P_i} \pi(p, m(i), F)$.

Definitions 1. *(i) A search market is defined by (m, q, v) satisfying Assumptions 1 and 2. (ii) A search market equilibrium is a strategy profile $f \in \mathcal{P}$ such that for all $i \in I$,*

$$\pi(f(i), m(i), F_f) \geq \pi(p, m(i), F_f) \tag{14}$$

for all $p \in P_i$, where $F_f \in \mathcal{D}$ is induced by f.

As Echenique and Edlin (2004) have shown, properly mixed strategy equilibria tend to be dynamically unstable in games with (strict) strategic complementarities, so we neglect mixed strategies throughout the paper. The requirement that consumers search optimally is already incorporated into the demand schedule (10) by construction, so we need not explicitly mention it in the definition of equilibrium.

3. Strategic Complementarities and Existence

An important feature of the above equilibrium concept is that players' payoffs depend on the cdf F of players' actions, as opposed to direct dependence on the strategy profile f itself. As such, we cannot appeal to standard existence results. The existence issue is therefore a nontrivial one and we begin this section with a brief discussion of previous results.

Reinganum (1979) considers a version of the model in this paper where consumers are identical (same search cost and downward-sloping demand), but firms have heterogeneous

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on $[0, \tilde{s}]$ because pdfs are only defined up to sets of measure zero. In particular, we are free to re-define $q(0) > 0$ as $q(0) = 0$. As our proofs show, the resulting discontinuity does not pose any insurmountable problems.

constant marginal costs. In that case, the equilibrium cdf of prices is directly induced by the cdf of firms' marginal costs, so existence is not really an issue. Rob (1985) considers the opposite extreme where firms are identical with zero marginal costs and consumers have perfectly inelastic demand for one unit of the good with infinite willingness to pay but heterogeneous search costs. Although his existence proof is ingenious, it does not take into account potentially downward-sloping demand and does not extend to the case of heterogeneous production costs. Carlson and McAfee (1983) also assume perfectly inelastic demand, but allow for bilateral heterogeneity in production and search costs. They provide an explicit solution for the equilibrium cdf of prices in the special case where q is uniform. Bénabou (1990, 1993) generalizes Carlson and McAfee (1983) to allow for downward-sloping demand and a broad class of search cost densities, but is unable to prove existence for reasons discussed in Bénabou (1990).

In Rauh (1997), consumers are endowed with a belief (probability measure on $\mathcal{D}$) about the cdf of prices based on a forecast of finitely many of its moments. In equilibrium, their forecast about the vector of moments is correct, but their beliefs need not be. In this case, existence follows from the main result in Rauh (2003) [also see the extension in Yu and Zhu (2005)]. Finally, Rauh (2007) uses nonstandard analysis and an appeal to Khan and Sun (1999) to prove existence in a general search model with heterogeneities in consumers' demand functions, search costs, and firms' cost functions, for a broad class of demand and cost functions. Although this approach yields existence under very general conditions, it sheds little light on the structure of the equilibrium set, price dispersion, or dynamic stability properties.

Stahl's (1989, 1996) model differs in some important respects from those discussed above. Although consumers still search sequentially, he allows for the case $Q(0) > 0$ where there is a positive mass of "shoppers" (consumers with zero search costs). Instead of a continuum, he assumes finitely many identical firms and he uses the more standard Nash equilibrium concept where players make correct conjectures about other players' strategies in equilibrium. To see the difference, consider the case where all firms play pure strategies. In his model, consumers would know which firm is charging the lowest price, whereas in our model they would only know the distribution of prices. In contrast, consumers would know the *realized* distribution of prices in our model when firms play mixed strategies, whereas in his model consumers would only know the mixed strategies themselves.[8] In the case where $Q(0) > 0$, Stahl (1996, proposition 2.1) shows that symmetric pure-strategy equilibria do not exist, but that symmetric mixed-strategy equilibria do (his theorem 3.1). Our existence proof is therefore more general than his in the sense that we allow for heterogeneous production costs, but less general in that we assume strategic complementarities as defined below. Indeed, Stahl's model might admit pure-strategy equilibria when firms differ with respect to their production costs, since the lowest cost firm would presumably undercut the others to capture the atom of shoppers.

Definition 2. *A search market is (i) (strictly) supermodular if* $\pi(p, m(i), F)$ *has (strictly) increasing differences or (ii) (strictly) log-supermodular if* $\log \pi(p, m(i), F)$ *is well-defined and has (strictly) increasing differences in* (p, F) *on* $P_i \times \mathcal{D}$ *for all* i.[9] *(iii) In either case, we say the search market is characterized by (strict) strategic complementarities.*

Proposition 4 below provides a necessary and sufficient condition for each form of strategic complementarity.

Proposition 4. *(i) If* $\tilde{s} < \overline{s}$ *then the search market is (strictly) log-supermodular iff*

$$\frac{q(\Gamma(p, F))}{N(p, F)} \tag{15}$$

is (strictly) decreasing on $\mathcal{D}$ *and (ii) (strictly) supermodular iff*

$$N(p, F)\left(\frac{\pi_p^M}{\pi^M}\right) - q(\Gamma(p, F))x(p) \tag{16}$$

is (strictly) increasing on $\mathcal{D}$ *for all* $p \in P_i$ *and all* i.

We now interpret the above conditions assuming $F > F'$ implies $N(p, F) > N(p, F')$ as in Proposition 3. Suppose firm i is charging the price p when its competitors raise their

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[8] This distinction applies whether there are finitely or infinitely many firms. In a symmetric mixed-strategy equilibrium with a continuum of firms, the realized price distribution will be the same as the equilibrium mixed-strategy assuming the law of large numbers. As an informational assumption prior to any specific equilibrium, however, the distinction remains.

[9] Let X and Y be partially ordered sets. A function $f : X \times Y \to \mathbf{R}$ exhibits *(strictly) increasing differences* on $X \times Y$ if $f(x, y) - f(x, y')$ is (strictly) increasing in x for all $y > y'$. Although $\log \pi$ is not well-defined for $p = m(i) \in P_i$, this is unimportant since clearly that is never a best response.

prices. If the search market exhibits strategic complementarities, this should induce i to raise its price as well (see Proposition 6 below). Since the denominator in (15) increases, the condition in (i) limits the size of any possible increase in the numerator. This implies that q cannot be decreasing too rapidly, since $\Gamma(p, F)$ is decreasing in F (see footnote 5). Intuitively, $\Gamma(p, F)$ is the search cost of a consumer who is just indifferent about buying from firm i. Condition (i) therefore limits the increase in the mass of these consumers, whom i would lose if it raised its price.

The condition in (ii) also limits any potential increase in $q(\Gamma(p, F))$, but now the restrictiveness of the condition depends on where firm i lies in the cdf of prices. If i's initial price p is low, then π_p^M/π^M may be relatively large and i already has a significant incentive to raise price to increase monopoly profits. In that case, $q(\Gamma(p, F))$ can increase substantially. In contrast, if p is already close to i's monopoly price then π_p^M/π^M is relatively small and the restriction on $q(\Gamma(p, F))$ will need to be relatively severe, since monopoly profit is already fairly flat with respect to price.

Although the conditions in Proposition 4 are intuitive, they seem extremely difficult to check. Our next result provides some simple sufficient conditions.

Proposition 5. *(i) If q is at least weakly increasing on $[0, \tilde{s}]$ and the relevant sufficient condition holds in Proposition 3 then the search market is strictly supermodular. (ii) It is (strictly) log-supermodular if $q'(s) \leq 0$ and $|q'(s)|^{1/2}$ $(<) \leq q(\tilde{s})$ for all $s \in [0, \tilde{s}]$.*

Both conditions are consistent with Proposition 4, which limits how negative the slope of q can be. In (i), q is weakly increasing, whereas the condition in (ii) establishes an explicit lower bound on $q' \leq 0$.

Examples of distributions that satisfy (i) include the uniform and truncated normal distributions as discussed earlier. In the latter case, the requirement that q is increasing on $[0, \tilde{s}]$ implies that $\tilde{s}$ is less than or equal to the mode, so the proportion of inactive searchers must be at least 50%. This is consistent with the marketing evidence discussed in the introduction, as well as some of the structural estimates in the empirical search literature discussed in the next section. The condition in (ii) requires q to be high and flat like the uniform distribution, which satisfies the strict version of the requirement. The uniform distribution therefore represents the canonical example of strategic complementarities, in the sense that Proposition 3 requires $q' < (1/V)q$ when q is increasing and Proposition 5 requires $|q'|^{1/2} < q(\tilde{s})$ when q is decreasing, so both conditions characterize distributions that are high and flat. Examples of distributions that violate (ii) include the Pareto and downward-sloping gamma distributions, including the exponential.

We now turn to individual firm comparative statics. We define a partial order on $\mathcal{P}$ as follows: $f \geq f'$ if $f(i) \geq f'(i)$ for all $i \in I$. In the appendix, we show that $\mathcal{P}$ with this relation is a complete lattice (see Theorem A).[10] Let ϕ be the best response correspondence

$$\phi(i, f) = \arg\max_{p \in P_i} \pi(p, m(i), F_f). \tag{17}$$

The proofs of the next two results are fairly standard.

Proposition 6. *If the search market exhibits strategic complementarities then the set* $\phi(i, f) \subseteq P_i$ *of best responses for firm* i *is nonempty and compact for all* $f \in \mathcal{P}$ *and all* i. *Moreover,* $\phi(i, f)$ *contains a largest* $\overline{\phi}(i, f)$ *and smallest* $\underline{\phi}(i, f)$ *element, both of which are measurable on* I *and increasing on* $\mathcal{P}$.

Given the strategy profile f, the profit-maximizing price for firm i may not be unique, but the set $\phi(i, f)$ of such prices will contain a maximum $\overline{\phi}(i, f)$ and minimum $\underline{\phi}(i, f)$ element. Strategic complementarities imply weak monotone comparative statics in the sense that an increase in prices implies $f \geq f'$, $\overline{\phi}(i, f) \geq \overline{\phi}(i, f')$, and $\underline{\phi}(i, f) \geq \underline{\phi}(i, f')$, so the set of best responses shifts up. If the profit-maximizing price is always unique then $\overline{\phi}(i, f) = \underline{\phi}(i, f)$ and we obtain monotone comparative statics in the conventional sense. Although the following result appears purely technical, it will play a crucial role in our analysis of price dispersion.

Theorem 1. *If a search market exhibits strategic complementarities then the equilibrium set is nonempty and has a largest* $\overline{f}$ *and smallest* $\underline{f}$ *element.*

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[10] A *lattice* $(X, \geq)$ is a partially ordered set such that $\inf\{x, x'\}$ and $\sup\{x, x'\}$ exist in X for all $x, x' \in X$. A lattice X is *complete* if $\sup S$ and $\inf S$ exist in X for all nonempty $S \subseteq X$.

4. Price Dispersion

One of the main objectives of the theoretical search literature has been to explain the existence of price dispersion in real-world markets, which has been observed everywhere from the modern internet [Baye, Morgan, and Scholten (2006)] to the Turkish bazaar [Caglayan, Filiztekin, and Rauh (2007)]. In Reinganum (1979), Rob (1985), and Stahl (1989, 1996), consumers search sequentially and dispersion occurs, respectively, because of downward-sloping demand and heterogeneous production costs; heterogeneous search costs and perfectly inelastic demand with infinite willingness to pay; and heterogeneous search costs and a positive atom of shoppers. In contrast, Burdett and Judd (1983) and Baye and Morgan (2001) show that dispersion can arise without any *ex ante* heterogeneities provided, respectively, consumers search nonsequentially or there exists an information clearinghouse controlled by a monopoly gatekeeper. In this section, we study the theoretical robustness of the explanations in Reinganum (1979) and Rob (1985).

Recall that the Diamond equilibrium is where all firms are identical and almost all of them charge the unique monopoly price. Lemma 1 below is similar to corollary 4.2 in Stahl (1996) and states that the Diamond equilibrium is the unique *symmetric* equilibrium when

$$q(0+) \equiv \lim_{s \to 0} q(s) \tag{18}$$

is sufficiently small.[11] Note that the lemma does not assume strategic complementarities. Let F_{p_0} denote the cdf of prices that assigns probability 1 to some price $p_0 \geq 0$.

Lemma 1. *Assume $q(0+)$ exists. (i) If demand x is perfectly inelastic at one unit on $[0, \overline{p}\,]$ and $q(0+) < 1/\overline{p}$ then $F_{\overline{p}}$ is the unique symmetric equilibrium. (ii) If firms are identical with marginal cost c and*

$$q(0+) < \frac{\pi_p^M}{x\pi^M} \tag{19}$$

for all $c < p < p_M(c)$ then $F_{p_M(c)}$ is the unique symmetric equilibrium.

In the case of perfectly inelastic demand with finite willingness to pay, the Diamond equilibrium obtains no matter how heterogeneous production and search costs are. This

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[11] Recall that q may have a discontinuity at zero so $q(0+)$ may be different from $q(0)$.

explains why Reinganum (1979) needed the assumption of downward-sloping demand and why Rob (1985) needed to assume infinite willingness to pay. If demand is properly downward-sloping, the Diamond equilibrium still obtains no matter how heterogeneous search costs are.

The intuition for uniqueness is as follows. Consider case (ii) and assume almost all firms are charging the price p_0, where $c < p_0 < p_M(c)$. If one firm were to raise its price above p_0, its monopoly profits would increase [see Assumptions 1 and (13)], but it would lose its customers with low search costs. When $q(0+)$ is relatively small, the former effect dominates and F_{p_0} cannot be an equilibrium. The result therefore confirms and extends the conventional wisdom about the importance of the behavior of the density of search costs near zero (but not *at* zero). We now come to the main result of this section.

Theorem 2. *If the search market exhibits strategic complementarities, firms are identical, and* $q(0+)$ *is sufficiently small in the sense of Lemma 1, then the Diamond Paradox obtains: the Diamond equilibria in Lemma 1 are the unique search market equilibria overall for their respective cases.*

Proof. Consider case (i) or (ii) in Lemma 1. According to that lemma, the Diamond equilibrium is the unique symmetric equilibrium. By Theorem 1, the set of search market equilibria has a largest $\overline{f}$ and smallest $\underline{f}$ element. In the case of $\overline{f}$, the equilibrium involves firms making their largest best response against $\overline{f}$ (see the proof of Theorem 1). Since firms are identical, their largest best response is the same, and $\overline{f}$ is a symmetric equilibrium. Since $\underline{f}$ is also a symmetric equilibrium, $\overline{f} = \underline{f}$ and the equilibrium is unique: the unique equilibrium is the Diamond equilibrium. ∎

Strategic complementarities therefore allow us to extend the conclusion of Lemma 1 above and corollary 4.2 in Stahl (1996) to include potential asymmetric equilibria: not only is the Diamond equilibrium the unique *symmetric* equilibrium, it is also the unique equilibrium overall.

We now compare our results with Rob (1985). We first note that Rob's assumption that demand is perfectly inelastic at one unit with infinite willingness to pay rules out the Diamond equilibrium *ex ante*. In his theorem 1, he shows that there are no other single-price equilibria if $q(0) = 0$ or if there exists a search cost $s \geq 0$ such that

$$Q(s) < \frac{sq(0)}{1 + q(0)}. \tag{20}$$

He then proves (his theorem 2) that an equilibrium exists if q and $(1 - Q)/q$ are bounded and $0 < q(0) < \infty$. If (20) holds, then this equilibrium must exhibit price dispersion. Rob's methodology, however, breaks down when $\overline{p}$ is finite. In that case, Lemma 1 shows that the Diamond equilibrium is indeed an equilibrium, so one cannot prove the existence of price dispersion by first ruling out all single-price equilibria and then proving existence.

Nevertheless, Rob's results are still useful for "reverse engineering" purposes. If we assume that firms are identical and $\overline{p} = \infty$ then we can apply his results to obtain a dispersed cdf F of prices and a constant $K \geq 0$ such that $\pi = K$ on the support of F and $\pi \leq K$ otherwise. If we then assume that $\overline{p}$ is greater than the supremum of the support of F then this will still be an equilibrium in our model. E.g., Rob (p. 501) constructs a specific q such that the uniform distribution on $[1, 2]$ is an equilibrium cdf of prices where π is increasing for prices in $[0, 1]$, equal to $K = 1/\log 2$ on $[1, 2]$, diminishing on $[2, 6]$, and then zero on $(6, \infty)$. If $\overline{p} > 2$ this will also be an equilibrium in our model.

Our Theorem 2 provides an intuitive necessary condition for the existence of such Rob-type dispersed equilibria in our model: the search market cannot exhibit strategic complementarities. This condition is intuitive, since it is expressed in terms of the strategic character of price competition. According to Rob's theorem 4, a necessary condition for an absolutely continuous equilibrium cdf of prices is that q must be decreasing [see his equation (14)]. Our Proposition 4 provides still further information: q must not only be decreasing, but decreasing sufficiently rapidly so that the search market does not exhibit strategic complementarities. Note that Theorem 2 extends further than Rob (1985) in that it also allows for downward-sloping demand.

We conclude that heterogeneous search costs are generally insufficient to generate price dispersion in search market models with sequential search. For dispersion to be a robust phenomenon, we need a positive atom of shoppers as in Stahl (1989, 1996) or some other source of heterogeneity such as heterogeneous production costs in Reinganum (1979) or heterogeneous sampling probabilities as in Hortaçsu and Syverson (2004). Indeed, our next result shows that a simple Reinganum-type condition is sufficient for *all* equilibria to be dispersed (existence is assured by Theorem 1). The proof is trivial and is omitted.

Lemma 2. *Let* I_R *be the set of all* i *such that there exists* $I' \subseteq I$ *of positive measure such that* $p_M(m(i)) < m(i')$ *for all* $i' \in I'$*. If* I_R *has positive measure then degenerate equilibria do not exist.*

In this case, price dispersion is not a purely informational phenomenon but is instead driven by imperfect information, downward-sloping demand, and heterogeneous production costs. As such, it is at least in part a *technological* phenomenon.

Empirical Search Literature[12]

We now discuss the empirical search literature on the structural estimation of search costs. The first question is: how high are search costs? If q is hill or bell-shaped like the truncated normal distribution, then one of the requirements for strict supermodularity in Proposition 5(i) is that the proportion of inactive searchers be at least 50%. The other issue concerns the importance of heterogeneities other than heterogeneous search costs. According to Theorem 2 above, the latter is not a robust source of price dispersion and even when a Rob-type dispersed equilibrium exists, Hopkins and Seymour (2002) suggest that it will be dynamically unstable and therefore will not be observed.

Hortaçsu and Syverson (2004) construct an equilibrium search model similar to the one in this paper, where consumers search sequentially with heterogeneous search costs. They assume finitely many vertically differentiated firms with heterogeneous production costs and sampling probabilities (in our model, two firms with the same price are equally likely to be sampled during the search process). Their data consists of price (basis points) and market share data from S&P 500 index funds, which are essentially homogeneous products although the fund companies themselves differ in several important respects. Their heterogeneous funds estimates (see their section V.C.) indicate that the median search cost is relatively low, the search cost density is decreasing as required in Rob (1985), and that product differentiation is indeed important.

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[12] I benefitted from extensive discussions with Matthijs Wildenbeest on this subject.

Hong and Shum (2006) propose structural estimation methodologies for both the Rob model and an extension of the Burdett and Judd (1983) nonsequential search model to the case of heterogeneous search costs. An advantage of their approach is that it can be implemented using only price data. As an illustration of their methods, they use online price data to estimate the search cost distributions for four well-known textbooks. Their nonparametric estimates for the nonsequential search model suggest that the proportion of inactive searchers is about 50%. For the Rob model, both the parametric (where the search cost distribution is assumed to be gamma) and nonparametric estimates reveal a decreasing search cost density where the proportion of inactive searchers is more than 50%.

Moraga-González and Wildenbeest (2008) develop a version of the Burdett and Judd nonsequential search model with finitely many firms and heterogeneous search costs and propose a new structural estimation method that improves upon Hong and Shum (2006) in several respects. Using online price data for computer memory chips, they find that the proportion of inactive searchers ranges from 22-30%, while the proportion of consumers who search only once or twice ranges between 61-90%. Furthermore, the estimated q can be increasing or decreasing (see their Figure 6, p. 24) and firm-level heterogeneity is also important, as found by Hortaçsu and Syverson (2004).

Moraga-González, Sándor, and Wildenbeest (2007) show how price data from distinct but related markets in the sense that consumers' valuations are different but the search cost distributions are the same can be used to improve the estimation procedure still further. Using online prices for 10 memory chips, they find that the proportion of inactive searchers ranges between 18-33% and the proportion of consumers who search once or twice between 55-77%. Wildenbeest (2007) considers a version of the Burdett and Judd model with finitely many firms, vertical product differentiation, and heterogeneous production and search costs. Using online price data for supermarkets, he finds that the proportion of inactive searchers is between 50-81% and that product differentiation explains 85-92% of observed price dispersion, with search frictions accounting for the rest. De los Santos (2007) can actually observe directly the number of searches per consumer in his data set, where the proportion of inactive searchers is between 52-85%.

The estimates in Hong and Shum (2006) and Wildenbeest (2007), as well as direct observation in De los Santos (2007), are therefore consistent with the sufficient condition for strict supermodularity in Proposition 5(i). Furthermore, Hortaçsu and Syverson (2004) and Moraga-González and Wildenbeest (2008) both find evidence that price dispersion is indeed driven by more than just heterogeneous search costs. On the other hand, the exact shape of the estimated q may depend on the theoretical model used. This is clearest in the case of the Rob model, which imposes a decreasing q on the data.

5. Dynamics

The equilibrium concept in Definitions 1 implies that consumers and firms act as if they know the cdf of prices in equilibrium. This requires a substantial amount of information and computational power, which seems at odds with the fundamental purpose of the search market literature, which is to study markets characterized by imperfect information. In particular, the equilibrium concept requires that the model (m, q, v) must be common knowledge: consumers and firms must know consumers' search costs, consumer demand, and firms' production costs. Given this knowledge, they must then be able to compute the equilibrium cdf of prices, which is something the theoretical literature itself has been unable to do except in certain special cases [see Carlson and McAfee (1983)].

In this paper, we assume consumers and firms are myopic and have enough information to piece together the cdf of prices that prevailed in the *previous* period. Although real-world agents may not even possess this amount of information, these assumptions are nevertheless orders of magnitude weaker than the common knowledge and computational requirements corresponding to our equilibrium concept. Given the cdf of prices from the previous period, consumers can calculate their reservation levels knowing only their own demand and search cost. To calculate their profit-maximizing prices, firms still need to know consumers' demand and search costs, but they do not need to know other firms' costs or be able to compute the equilibrium cdf of prices.

A *best response orbit* is a sequence $\{f_n\} \subseteq \mathcal{P}$ of strategy profiles such that $f_{n+1}(i) \in \phi(i, f_n)$ for all $i \in I$ and $n \geq 0$ for some initial condition $f_0 \in \mathcal{P}$. In other words, consumers' reservation levels and firms' profit-maximizing prices are best responses against the cdf of prices from the previous period.[13] Given two strategy profiles a and b, we can define an interval in $\mathcal{P}$ as follows:

$$[a, b] = \{f \in \mathcal{P} \,|\, a \leq f \leq b\}. \tag{21}$$

We say that a sequence $\{f_n\} \subseteq \mathcal{P}$ *approaches* $[a, b]$ if there exist two sequences $\{\underline{f}_n\}$ and $\{\overline{f}_n\}$ in $\mathcal{P}$ such that $\underline{f}_n \leq f_n \leq \overline{f}_n$ for all $n \geq 0$ and $\underline{f}_n \uparrow a$ and $\overline{f}_n \downarrow b$ almost uniformly.[14] In other words, $\{f_n\}$ is being "squeezed" towards $[a, b]$ from above and below by two sequences in $\mathcal{P}$ converging monotonically and almost uniformly to its endpoints. Let

$$\begin{aligned} \mathcal{P}^+ &= \{f \in \mathcal{P} \,|\, f \geq \overline{\Psi}(f)\} \\ \mathcal{P}^- &= \{f \in \mathcal{P} \,|\, f \leq \underline{\Psi}(f)\}, \end{aligned} \tag{22}$$

where $\overline{\Psi} : \mathcal{P} \to \mathcal{P}$ is defined by $f \mapsto \overline{\phi}(i, f)$ and similarly for $\underline{\Psi}$.

Milgrom and Roberts (1990) and Vives (1990) have shown that supermodular games with finitely many players have nice stability properties. Theorem 3 below shows that, to some extent, this extends to equilibrium search models with a continuum of firms. Given the above definitions, the proof involves minor adaptations of the argument for theorem 5.1 in Vives (1990, p. 313) to account for the infinite-dimensional and measure-theoretic aspects of the current context.

Theorem 3. *Assume strict strategic complementarities and that if the cdf of prices is the same in periods n and $n+1$ then firms set the same prices at $n+2$ as they did at $n+1$.*

(i) Every best response orbit approaches $[\underline{f}, \overline{f}]$, where $\underline{f}$ is the smallest search market equilibrium and $\overline{f}$ is the largest.

(ii) For any best response orbit $\{f_n\}$ with initial condition in $\mathcal{P}^+$ ($\mathcal{P}^-$), there exists a search market equilibrium f such that $f_n \downarrow f$ ($f_n \uparrow f$).

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[13] Recall that N in (10) incorporates optimal consumer search by construction. Also recall that f_n enters $\phi(i, f_n)$ via the induced cdf F_{f_n} of prices as in (13). Our notation is therefore mathematically correct but economically slightly misleading, since it suggests that consumers and firms act as if they know f_n when they really act as if they know F_{f_n}.

[14] Let f and f_n for all $n \geq 1$ be measurable functions $I \to \mathbf{R}$ and λ denote Lebesgue measure on $\mathbf{R}$. We say that $f_n \to f$ *almost uniformly* if for all $\epsilon > 0$ there exists $I' \subseteq I$ such that $\lambda(I') < \epsilon$ and $f_n \to f$ uniformly on the complement of I' in I. See Folland (1999, p. 60). We write $f_n \uparrow f$ if $f_n \leq f_{n+1}$ for all $n \geq 1$ and $f_n \to f$ almost uniformly. The definition for $f_n \downarrow f$ is similar.

(iii) If the search market equilibrium is unique, then every best response orbit approaches it.

The first result states that all best response orbits starting from any initial condition approach the interval $[\underline{f}, \overline{f}]$ in $\mathcal{P}$ determined by the largest and smallest search market equilibria. It follows that if the best response dynamics exhibit any interesting regularities (limiting behaviors such as convergence to a fixed point or a periodic cycle), then they will occur arbitrarily close to that interval. In (ii), we show that orbits with initial conditions in $\mathcal{P}^+$ ($\mathcal{P}^-$) converge monotonically downward (upward) to search market equilibria. In particular, in the proof we show that the orbit starting from monopolistic expectations $p_M(m(i)) \in \mathcal{P}^+$ converges monotonically downward to $\overline{f}$, while the orbit starting from perfectly competitive expectations $m(i) \in \mathcal{P}^-$ converges monotonically upward to $\underline{f}$, so the result is not vacuous. According to (iii), any unique search market equilibrium is approached from all initial conditions (global stability), where "approach" is a strong form of convergence involving almost uniform squeezing from above and below. In particular, the Diamond equilibrium is globally stable when the Diamond Paradox obtains as originally discovered by Diamond (1971).

On the other hand, the same result can be applied to show that there exist *price dispersed* equilibria that are globally stable. E.g., Bénabou (1990) provides weak sufficient conditions for the existence of a unique price dispersed equilibrium when q is uniform. In such equilibria, price dispersion is a product of bilateral heterogeneity in both production and search costs. Since the uniform distribution supports strict strategic complementarities (see Proposition 5), our result shows that the unique price dispersed equilibria derived in his paper are globally stable for the best response dynamics. We can therefore interpret them as long run equilibria where consumers and firms act as if they know the equilibrium cdf of prices which can be approximated arbitrarily closely by short run "equilibria" where consumers' reservation levels and firms' prices are best responses against the cdf of prices that prevailed in the previous period.

We now compare our Theorem 3 with the results in Hopkins and Seymour (2002), who consider a much more general class of learning rules that includes fictitious play, which is analogous to best response dynamics. For complexity reasons, they are forced to study what are essentially finite-dimensional dynamics whereas in this paper we are able to consider fully infinite-dimensional dynamics because of our assumption of strict strategic complementarities. They show that the price dispersed mixed-strategy equilibria in Burdett and Judd (1983) and Varian (1980) are generally dynamically unstable when both consumers and firms learn. They also conjecture (p. 1163) the same for the dispersed equilibria in Rob (1985) and other search models with the "rock-scissors-paper" property such as Stahl (1989, 1996). Although these models can explain price dispersion with little or no heterogeneity, their dispersed equilibria tend to be dynamically unstable and therefore should not be observed. In contrast, the equilibria in Reinganum (1979) and Bénabou (1990) are generally stable, although they require heterogeneous production costs, a technological issue. Recall that in this paper we did not consider mixed strategies because Echenique and Edlin (2004) have shown that mixed-strategy equilibria are unstable in strict supermodular games.

6. Conclusion

In this paper, we applied concepts and tools from supermodular game theory to search models with sequential search, including Reinganum (1979), Carlson and McAfee (1983), Rob (1985), Stahl (1989, 1996), and Bénabou (1990, 1993). We identified necessary and sufficient conditions for the relevant pricing game to exhibit strategic complementarities, as well as simple sufficient conditions involving the hazard rate of the search cost density when the latter is monotonic (increasing or decreasing) over the range of search costs corresponding to active searchers. For the specific case of an increasing density over that interval, the proportion of inactive searchers must be at least 50%, which is consistent with the structural estimates in Hong and Shum (2006) and Wildenbeest (2007), as well as the directly observed search behavior in De los Santos (2007).

Given strategic complementarities, we proved existence of search market equilibrium and that the Diamond Paradox obtains with identical firms and when the search cost density is small near zero, no matter how heterogeneous consumers' search costs are. A basic insight of the paper is that price dispersion is therefore inherently incompatible with strategic complementarities, where strategic variables tend to be set at comparable levels.

To generate price dispersion as a robust phenomenon, we either need an atom of shoppers as in Stahl (1989, 1996) or some additional form of heterogeneity such as heterogeneous production costs as in Reinganum (1979) or heterogeneous sampling probabilities as in Hortaçsu and Syverson (2004). The latter paper and Moraga-González and Wildenbeest (2008) both find evidence that firm-level heterogeneities are indeed important. In our model, we showed that a simple Reinganum-type condition ensures that all equilibria are price dispersed.

Finally, we showed that sequential search markets with strategic complementarities have nice stability properties. In particular, when consumers and firms are identical except for heterogeneous search costs, the Diamond Paradox is a robust phenomenon that is globally stable in the best response dynamics. In contrast, price dispersion becomes a robust phenomenon under bilateral heterogeneity and the unique price dispersed equilibria in Reinganum (1979), Carlson and McAfee (1983), and Bénabou (1990, 1993) are all globally stable. We can therefore interpret them as long run equilibria where consumers and firms act as if they know the equilibrium distribution of prices.

Appendix

Proof of Proposition 1

We first note that $\Gamma : [0, \overline{p}\,] \times \mathcal{D} \to \mathbf{R}_+$ in (2) is well-defined, since the integrand is almost everywhere continuous and bounded from above by $x(0)$. It is therefore Riemann and Lebesgue integrable. Claim (i) follows from theorem 5.5 and proposition 6.5 in Haaser and Sullivan (1991, p. 237, 240) and (ii) is obvious. Since F first-order stochastically dominates F_m for all $F \in \mathcal{D}$,

$$\begin{aligned} 0 \leq \Gamma(r, F) \leq \Gamma(\overline{p}, F) &= \int_0^{\overline{p}} x(p)\, F(p)\, dp \\ &\leq \int_0^{\overline{p}} x(p)\, F_m(p)\, dp \equiv \tilde{s} \qquad (A.1) \end{aligned}$$

for all $0 \leq r \leq \overline{p}$, so (iii) follows. ▮

Before proving Proposition 2, we remark that G is absolutely continuous on $[0, \overline{p})$ because Q is absolutely continuous and $\Gamma(r, F)$ is increasing and absolutely continuous by Proposition 1. The calculation in (6) is therefore valid.

Proof of Proposition 2

Recall that the product of a bounded measurable function and an integrable function is integrable. Now, $q(\Gamma(r, F))$ is bounded and continuous in r except possibly at $r = z_F$ and at the point where $\Gamma = \overline{s}$ if $\overline{s}$ is finite. It follows that $q(\Gamma(r, F))$ is bounded measurable. Since $x(r)$ is integrable, the integral in (10) is well-defined. The rest follows from theorem 5.5 and proposition 6.5 in Haaser and Sullivan (1991, p. 237, 240). ▌

Proof of Proposition 3

Let $F > F'$ and define $F_t = tF + (1-t)F'$ for all $t \in [0, 1]$. Note that $F_t \in \mathcal{D}$ for all $t \in [0, 1]$. If we can show that $N(p, F_t)$ is strictly increasing in t on $[0, 1]$ then

$$N(p, F) = N(p, F_1) > N(p, F_0) = N(p, F'). \qquad (A.2)$$

Since $\Gamma = 0$ on $[0, z_F]$ and $q(0) = 0$,

$$N(p, F_t) = \int_{\max\{p, z_F\}}^{\overline{p}} q(\Gamma(r, F_t)) x(r)\, dr + 1 - Q(\Gamma(\overline{p}, F_t)). \qquad (A.3)$$

We first consider the case $\max\{p, z_F\} < \overline{p}$. On $(\max\{p, z_F\}, \overline{p})$ we have

$$0 < \Gamma(r, F_t) < \tilde{s} < \overline{s}, \qquad (A.4)$$

where q is differentiable. Taking the derivative of $N(p, F_t)$ with respect to $t \in (0, 1)$,

$$\int_{\max\{p, z_F\}}^{\overline{p}} q'(\Gamma(r, F_t)) h(r) x(r)\, dr - q(\Gamma(\overline{p}, F_t)) h(\overline{p}), \qquad (A.5)$$

where

$$h(r) = \frac{\partial \Gamma(r, F_t)}{\partial t} = \int_0^r x(p)[F(p) - F'(p)]\, dp. \qquad (A.6)$$

Note that $h \leq 0$ and is decreasing on $[0, \overline{p}]$ with $h(\overline{p}) < 0$ since $F > F'$. If $q' \leq 0$ on $(0, \tilde{s})$ then (A.5) is unambiguously positive and we are done. We therefore assume $q' \geq 0$ on

$(0, \tilde{s})$ from now on. Since $h(\overline{p}) \leq h(r)$, a sufficient condition for the expression in (A.5) to be positive is

$$h(\overline{p}) \int_{\max\{p, z_F\}}^{\overline{p}} q'(\Gamma(r, F_t))x(r)\, dr - q(\Gamma(\overline{p}, F_t))h(\overline{p}) > 0. \tag{A.7}$$

This reduces to

$$\int_{\max\{p, z_F\}}^{\overline{p}} q'(\Gamma(r, F_t))x(r)\, dr < q(\Gamma(\overline{p}, F_t)) \tag{A.8}$$

because $h(\overline{p}) < 0$. We now assume $q' \leq kq$ for some nonnegative constant k to obtain the sufficient condition

$$k \int_{\max\{p, z_F\}}^{\overline{p}} q(\Gamma(r, F_t))x(r)\, dr < q(\Gamma(\overline{p}, F_t)). \tag{A.9}$$

Since q is increasing on $[0, \tilde{s}]$,

$$kq(\Gamma(\overline{p}, F_t)) \int_{\max\{p, z_F\}}^{\overline{p}} x(r)\, dr < q(\Gamma(\overline{p}, F_t)). \tag{A.10}$$

A sufficient condition is therefore

$$k \int_{\max\{p, z_F\}}^{\overline{p}} x(r)\, dr < 1 \tag{A.11}$$

because $q(\Gamma(\overline{p}, F_t)) > 0$. The integral in (A.11) is strictly less than V because

$$\underline{c} \leq m(i) < p \leq \max\{p, z_F\} \tag{A.12}$$

for all $p \in P_i$ so it suffices to choose $k = 1/V$. The case $\max\{p, z_F\} = \overline{p}$ is simpler because the derivative of $1 - Q(\Gamma(\overline{p}, F_t))$ with respect to $t \in (0, 1)$ is $-q(\Gamma(\overline{p}, F_t))h(\overline{p}) > 0$. ▌

Proof of Proposition 4

We first prove (i). The assumption $\tilde{s} < \overline{s}$ implies a positive atom of consumers with reservation level $\overline{p}$ for all $F \in \mathcal{D}$. It follows that N and $\pi^M N$ are positive on $P_i \times \mathcal{D}$, so $\log \pi$ in (13) is well-defined. Let $F > F'$ in $\mathcal{D}$. To show (strictly) increasing differences, we must show that

$$\log \pi(p, m(i), F) - \log \pi(p, m(i), F') = \log N(p, F) - \log N(p, F') \tag{A.13}$$

is (strictly) increasing on P_i. According to Proposition 2, N is differentiable on P_i except at most two points. Now, theorem 5.11 in Rudin (1976, p. 108) states that a continuous real-valued function defined on an interval is (strictly) increasing if its derivative is (positive) nonnegative in the interior. If necessary, we divide P_i into three subintervals where N is differentiable in the interiors. Differentiating (A.13), (strictly) increasing differences is equivalent to

$$-\frac{q(\Gamma(p,F))x(p)}{N(p,F)}+\frac{q(\Gamma(p,F'))x(p)}{N(p,F')}(>)\geq 0 \tag{A.14}$$

or

$$\frac{q(\Gamma(p,F'))}{N(p,F')}(>)\geq\frac{q(\Gamma(p,F))}{N(p,F)}, \tag{A.15}$$

for all $p\in P_i$ and all i. This completes the proof of (i). To prove (ii), we note that

$$\pi(p,m(i),F)-\pi(p,m(i),F')=\pi^M(p,m(i))[N(p,F)-N(p,F')] \tag{A.16}$$

and differentiate as before. ∎

Proof of Proposition 5

To prove (i), we must show that (A.16) is strictly increasing on P_i. Since π^M is strictly increasing on P_i by Assumptions 1 and $N(p,F)-N(p,F')>0$ on $P_i\times\mathcal{D}$ by Proposition 3, all we need to do is show that the latter is at least weakly increasing on P_i. Differentiating,

$$-q(\Gamma(p,F))x(p)+q(\Gamma(p,F'))x(p)\geq 0 \quad\iff\quad q(\Gamma(p,F'))\geq q(\Gamma(p,F)). \tag{A.17}$$

Since $\Gamma(p,F)\leq\Gamma(p,F')$ (see footnote 5), this holds when q is at least weakly increasing on $[0,\tilde{s}]$. To prove (ii), we define F_t as in the proof of Proposition 3 and consider $\max\{p,z_F\}<\overline{p}$ since the case $\max\{p,z_F\}=\overline{p}$ is simpler. According to Proposition 4(i), the search market is (strictly) log-supermodular iff q/N is (strictly) decreasing in $t\in[0,1]$ for all $p\in P_i$. Differentiating,

$$N(p,F_t)q'(\Gamma(p,F_t))h(p)-q(\Gamma(p,F_t))\int_{\max\{p,z_F\}}^{\overline{p}}q'(\Gamma(r,F_t))h(r)x(r)\,dr$$

$$+\,q(\Gamma(p,F_t))q(\Gamma(\overline{p},F_t))h(\overline{p}). \tag{A.18}$$

Since $q' \leq 0$ and $h \leq 0$, the first two terms in (A.18) are nonnegative, while the third is negative. A sufficient condition is therefore

$$N(p, F_t)q'(\Gamma(p, F_t))h(p) + q(\Gamma(p, F_t))q(\Gamma(\overline{p}, F_t))h(\overline{p})\,(<) \leq 0. \tag{A.19}$$

In the proof of Proposition 3, we showed that h is decreasing and $h(\overline{p}) < 0$, so the previous condition reduces to

$$N(p, F_t)q'(\Gamma(p, F_t)) + q(\Gamma(p, F_t))q(\Gamma(\overline{p}, F_t))\,(>) \geq 0. \tag{A.20}$$

Since Γ is increasing (see Proposition 1), q is decreasing, and $N \leq 1$,

$$q'(\Gamma(p, F_t)) + q(\tilde{s})^2\,(>) \geq 0, \tag{A.21}$$

which completes the proof. ▮

The following result is similar to lemma 6.1 in Vives (1990, p. 315). A lattice X is *conditionally complete* if $\sup S$ and $\inf S$ exist in X for all nonempty and bounded $S \subseteq X$. For example, $\mathbf{R}$ is conditionally complete. Formally, we identify strategy profiles which are equal almost everywhere with respect to Lebesgue measure, so $\mathcal{P}$ is the space of all equivalence classes of such profiles.

Theorem A. *$\mathcal{P}$ is a complete lattice.*

Proof. Since the functions max and min are continuous, $\mathcal{P}$ is a lattice. Let $S \subseteq \mathcal{P}$ be nonempty. Welland (1964, p. 267-8) shows that the space L_1 of equivalence classes of real-valued, measurable, and absolutely summable functions $I \to \mathbf{R}$ is conditionally complete. Each $f \in \mathcal{P}$ is bounded and measurable, so $S \subseteq L_1$. Since p_M is increasing on a compact interval $[\underline{c}, \overline{c}]$, it is bounded and almost everywhere differentiable, hence both Riemann and Lebesgue integrable [Haaser and Sullivan (1991, theorem 3.7, p. 228)]. A composition of measurable functions is measurable, so $p_M(m(i))$ is bounded, measurable, and absolutely summable on I. Since S is bounded from above by $p_M \circ m \in L_1$ and from below by $m \in L_1$ and L_1 is conditionally complete, it follows that $\sup S$ and $\inf S$ exist in L_1. Since $m \leq \inf S \leq \sup S \leq p_M \circ m$, we have $\inf S, \sup S \in \mathcal{P}$ and we are done. ▮

Proof of Proposition 6

Fix $i \in I$. Each $f \in \mathcal{P}$ induces an $F \in \mathcal{D}$. For all $F \in \mathcal{D}$, π and $\log \pi$ are continuous and trivially supermodular on P_i. By theorem 2.4 and its corollary in Vives (1999, p. 30), $\phi(i, f)$ is nonempty and compact, with largest $\overline{\phi}(i, f)$ and smallest $\underline{\phi}(i, f)$ elements. Now fix $f \in \mathcal{P}$ and the $F_f \in \mathcal{D}$ it induces. By the Measurable Maximum Theorem in Aliprantis and Border (1999, p. 570), the best response correspondence ϕ is measurable on I. Applying the Measurable Maximum Theorem again, with p as the objective function and ϕ as the constraint correspondence, we see that $\overline{\phi}$ is measurable on I. A similar argument with $-p$ as the objective function shows that $\underline{\phi}$ is measurable on I. The fact that the extremal best responses are increasing on $\mathcal{P}$ follows from $f \geq f' \Rightarrow F_f \geq F_{f'}$, increasing differences for π or $\log \pi$ on $P_i \times \mathcal{D}$, and theorem 2.4 and its corollary in Vives (1999, p. 30), which completes the proof. ▌

Proof of Theorem 1

According to Proposition 6, $\overline{\phi}(i, f)$ is measurable on I and the map $\overline{\Psi} : \mathcal{P} \to \mathcal{P}$ defined by $f \mapsto \overline{\phi}(i, f)$ is increasing. We can therefore apply the Tarski fixed point theorem [theorem 2.2 in Vives (1999, p. 20)] to obtain the largest fixed point $\overline{f}$ of $\overline{\Psi}$ satisfying

$$\overline{f} = \sup\{f \in \mathcal{P} \mid \overline{\Psi}(f) \geq f\}. \qquad (A.22)$$

Let f be any Nash equilibrium. Since $\overline{\phi}(i, f) \geq f(i)$ for all $i \in I$, it follows that $\overline{\Psi}(f) \geq f$ and hence $\overline{f} \geq f$. The proofs for $\underline{f}$ are similar. ▌

Proof of Lemma 1

It is obvious that $F_{\overline{p}}$ and $F_{p_M(c)}$ are equilibria in their respective cases. To prove (ii), let $c \leq p_0 < p_M(c)$. We first derive the expression for the right-hand derivative of π at p_0. By Proposition 2, N is decreasing on $[0, \overline{p}]$ for all $F \in \mathcal{D}$, so it is everywhere differentiable from the right. For $\epsilon > 0$ sufficiently small, $p_0 < p_0 + \epsilon < p_M(c) \leq \overline{p}$ and

$$\frac{N(p_0 + \epsilon, F_{p_0}) - N(p_0, F_{p_0})}{\epsilon} = -\frac{\int_{p_0}^{p_0+\epsilon} q\left(\int_{p_0}^{r} x(p)\, dp\right) x(r)\, dr}{\epsilon}. \qquad (A.23)$$

Next, we note that

$$0 < \int_{p_0}^{p_0+\epsilon} x(p)\,dp < \overline{s} \tag{A.24}$$

for $\epsilon > 0$ sufficiently small. In (A.23), the numerator and denominator $\rightarrow 0$ as $\epsilon \rightarrow 0$, so we apply L'Hospital's Rule to get

$$\lim_{\epsilon \to 0} \frac{N(p_0+\epsilon, F_{p_0}) - N(p_0, F_{p_0})}{\epsilon} = -\lim_{\epsilon \to 0} q\left(\int_{p_0}^{p_0+\epsilon} x(p)\,dp\right) x(p_0+\epsilon). \tag{A.25}$$

In differentiating the numerator, we used the assumption that q is continuous on $(0, \overline{s})$ and (A.24). Since x is continuous and by hypothesis $q(0+)$ exists,

$$\lim_{\epsilon \to 0} q\left(\int_{p_0}^{p_0+\epsilon} x(p)\,dp\right) x(p_0+\epsilon) = q(0+)x(p_0). \tag{A.26}$$

We now take the right-hand derivative of π at p_0 to get

$$-\pi^M q(0+)x(p_0) + N(p_0, F_{p_0})\pi_p^M. \tag{A.27}$$

Since consumers are evenly spread across firms, $N(p_0, F_{p_0}) = 1$. The expression in (A.27) is therefore positive if

$$-\pi^M q(0+)x(p_0) + \pi_p^M > 0. \tag{A.28}$$

We have $\pi_p^M(p_0, c) > 0$ since $c \leq p_0 < p_M(c)$. If $p_0 = c$, then $\pi^M = 0$ and (A.28) holds. If $c < p_0 < p_M(c)$, (A.28) boils down to (19). Hence, almost all firms prefer to raise price. The proof for (i) is similar but easier. ▌

A correspondence ϕ from a partially ordered set X into a lattice Y is *strongly increasing* if $x > x'$ implies $y \geq y'$ for all $y \in \phi(x)$ and $y' \in \phi(x')$. In other words, a correspondence is strongly increasing iff all its selections are increasing.

Proof of Theorem 3

We first prove (ii). Let $f_0 \in \mathcal{P}^+$. If firms always choose their largest best response, the corresponding orbit is $\{\overline{f}_n\}_{n=0}^{\infty}$ defined by $\overline{f}_{n+1} = \overline{\Psi}(\overline{f}_n)$ and $\overline{f}_0 = f_0$. By definition of $\mathcal{P}^+$, $\overline{f}_1 = \overline{\Psi}(f_0) \leq f_0$. Since $\overline{\Psi}$ is increasing, $\overline{f}_{n+1} \leq \overline{f}_n$ for all $n \geq 0$. If firms always choose their smallest best response, the corresponding orbit is $\{\underline{f}_n\}$ defined by $\underline{f}_{n+1} = \underline{\Psi}(\underline{f}_n)$.

Since $\underline{f}_1 = \underline{\Psi}(f_0) \leq \overline{\Psi}(f_0) \leq f_0$ and $\underline{\Psi}$ is also increasing, $\underline{f}_{n+1} \leq \underline{f}_n$ for all $n \geq 0$. Now take any best response orbit $\{f_n\} \subseteq \mathcal{P}$ with initial condition f_0. Since $f_1(i) \in \phi(i, f_0)$ for all i, $f_1 \leq \overline{\Psi}(f_0) \leq f_0$. If $f_1 = f_0$ for almost all i, then $F_{f_1} = F_{f_0}$ and $f_2 = f_1$ by hypothesis. Otherwise, $F_{f_1} < F_{f_0}$ and by hypothesis π or $\log \pi$ has strictly increasing differences on $P_i \times \mathcal{D}$ for all i. By theorem 2.3(iv) in Vives (1999, p. 26), firms' best response correspondences are strongly increasing on $\mathcal{D}$. Hence, $f_2 \leq f_1$ and $f_{n+1} \leq f_n$ for all $n \geq 0$ follows by induction. The best response orbit is therefore decreasing in all cases.

Let $\{f_n\}$ be any decreasing best response orbit. By the monotone convergence theorem [Lang (1993, theorem 5.5, p. 139)], there exists an integrable $f : I \to \mathbf{R}$ such that $f_n(i) \to f(i)$ almost everywhere on I. Since $m(i) \leq f_n(i) \leq p_M(m(i))$ for all i and $n \geq 0$, the same holds for f almost everywhere. By Egoroff's theorem [Folland (1999, p. 62], the convergence $f_n \to f$ is almost uniform. We now show that f is an equilibrium, once we have fixed it on a set of measure zero. First, fix any $i \in I$ in the subset of I of full measure such that $f_n(i) \to f(i)$. According to Dudley (2002, p. 288, 292, 295), $F_{f_n} \to F_f$ when $\mathcal{D}$ is endowed with the topology of weak convergence. Furthermore, π and $\log \pi$ are continuous on $P_i \times \mathcal{D}$ by Proposition 3 in Rauh (2007). Since $f_n(i) \to f(i)$, $F_{f_n} \to F_f$, and $f_n(i) \in \phi(i, f_{n-1})$ for all $n \geq 1$, we have $f(i) \in \phi(i, f)$ because π and $\log \pi$ are continuous. We are then free to alter f for the other firms (who comprise a set of measure zero) so that they are making best responses against f. After doing so, f is an equilibrium. The proof for $f \in \mathcal{P}^-$ is similar.

We now prove (i). Let $\{\overline{f}_n\}$ be defined as before, starting from $\overline{f}_0 = p_M \circ m$ and $\{\underline{f}_n\}$ starting from $\underline{f}_0 = m$. Clearly, $\overline{f}_0 \in \mathcal{P}^+$ and $\underline{f}_0 \in \mathcal{P}^-$. From (ii), there exists an equilibrium $\overline{f}$ ($\underline{f}$) such that $\overline{f}_n \downarrow \overline{f}$ ($\underline{f}_n \uparrow \underline{f}$) and convergence is also pointwise almost everywhere. We now show that $\overline{f}$ is the largest equilibrium, recalling that we identify functions which differ at most on a set of measure zero. Fix any equilibrium f. Clearly, $\overline{f}_0 = p_M \circ m \geq f$. Moreover, $\overline{f}_n \geq f$ implies

$$\overline{f}_{n+1} = \overline{\Psi}(\overline{f}_n) \geq \overline{\Psi}(f) \geq f, \tag{A.29}$$

so $\overline{f}_n \geq f$ for all $n \geq 0$. It follows that $\overline{f} \geq f$ almost everywhere, so we are done. A similar argument shows that $\underline{f}$ is the smallest equilibrium. Now take any best response orbit $\{f_n\}$ with initial condition f_0. Clearly, $\underline{f}_0 \leq f_0 \leq \overline{f}_0$. Furthermore,

$$\underline{f}_1 = \underline{\Psi}(\underline{f}_0) \leq \underline{\Psi}(f_0) \leq f_1 \leq \overline{\Psi}(f_0) \leq \overline{\Psi}(\overline{f}_0) = \overline{f}_1. \tag{A.30}$$

By induction, $\underline{f}_n \leq f_n \leq \overline{f}_n$, so we are done. ▌

References

Aliprantis, C.D., Border, K.C., 1999. Infinite Dimensional Analysis. Springer-Verlag, New York.

Baye, M.R., Morgan, J., 2001. Information gatekeepers on the internet and the competitiveness of homogeneous product markets. Amer. Econ. Rev. 91, 454-74.

Baye, M.R., Morgan, J., Scholten, P., 2006. Information, search and price dispersion. In: Hendershott, T. (Ed.), Handbook on Economics and Information Systems. Elsevier, Amsterdam / New York, pp. 323-375.

Bénabou, R., 1990. Search market equilibrium, bilateral heterogeneity, and repeat purchases. MIT working paper.

Bénabou, R., 1993. Search market equilibrium, bilateral heterogeneity, and repeat purchases. J. Econ. Theory 60, 140-158.

Bénabou, R., Gertner, R., 1993. Search with learning from prices: does increased uncertainty lead to higher markups? Rev. Econ. Stud. 60, 69-94.

Burdett, K., Judd, K.L., 1983. Equilibrium price dispersion. Econometrica 51, 955-69.

Caglayan, M., Filiztekin, A., Rauh, M.T., 2008. Inflation, price dispersion, and market structure. Europ. Econ. Rev. forthcoming.

Carlson, J., McAfee, P., 1983. Discrete equilibrium price dispersion. J. Polit. Econ. 91, 480-493.

Dana, J.D. Jr., 1994. Learning in an equilibrium search model. Int. Econ. Rev. 35, 745-71.

De los Santos, B.I., 2007. Consumer search on the internet. Working paper.

Diamond, P.A., 1971. A model of price adjustment. J. Econ. Theory 3, 156-68.

Dudley, R.M., 2002. Real Analysis and Probability. Cambridge University Press, Cambridge.

Echenique, F., Edlin, A., 2004. Mixed equilibria are unstable in games of strategic complements. J. Econ. Theory 118, 61-79.

Folland, G.B., 1999. Real Analysis. John Wiley and Sons, Inc., New York.

Haaser, N.B., Sullivan, J.A., 1991. Real Analysis. Dover, New York.

Hawkins, D.I., Best, R.J., Coney, K.A., 1995. Consumer Behavior: Implications for Marketing Strategy. Irwin, Chicago.

Hong, H., Shum, M., 2006. Using price distributions to estimate search costs. RAND J. Econ. 37, 257-75.

Hopkins, E., Seymour, R.M., 2002. The stability of price dispersion under seller and consumer learning. Int. Econ. Rev. 43, 1157-1190.

Hortaçsu, A., Syverson, C., 2004. Product differentiation, search costs, and competition in the mutual fund industry: a case study of S&P 500 index funds. Quart. J. Econ. 119, 403-456.

Janssen, M.C.W., Moraga-González, J.L., 2004. Strategic pricing, consumer search, and the number of firms. Rev. Econ. Stud. 71, 1089-1118.

Janssen, M.C.W., Moraga-González, J.L., Wildenbeest, M.R., 2005. Truly costly sequential search and oligopolistic pricing. Intl. J. Ind. Org. 23, 451-66.

Janssen, M.C.W., Parakhonyak, A., 2008. Consumer search with costly recall. Working paper.

Khan, M.A., Sun, Y.N., 1999. Non-cooperative games on hyperfinite Loeb spaces. J. Math. Econ. 31, 455-492.

Lang, S., 1993. Real and Functional Analysis. Springer-Verlag, New York.

Lippman, S.A., McCall, J.J., 1981. The Economics of Uncertainty. In: Arrow, K.J., Intriligator, M.D. (Eds.), Handbook of Mathematical Economics, Vol. I. North-Holland, Amsterdam, pp. 211-227.

Milgrom, P., Roberts, J., 1990. Rationalizability, learning, and equilibrium in games with strategic complementarities. Econometrica 58, 1255-77.

Moraga-González, J.L., Wildenbeest, M.R., 2008. Maximum likelihood estimation of search costs. Europ. Econ. Rev. forthcoming.

Moraga-González, J.L., Sándor, Z., Wildenbeest, M.R., 2007. Nonparametric estimation of the costs of non-sequential search. Working paper.

Rauh, M.T., 1997. A model of temporary search market equilibrium. J. Econ. Theory 77, 128-153.

Rauh, M.T., 2003. Non-cooperative games with a continuum of players whose payoffs depend on summary statistics. Econ. Theory 21, 901-906.

Rauh, M.T., 2004. Wage and price controls in the equilibrium sequential search model. Europ. Econ. Rev. 48, 1287-1300.

Rauh, M.T., 2007. Nonstandard foundations of equilibrium search models. J. Econ. Theory 132, 518-529.

Reinganum, J.F., 1979. A simple model of equilibrium price dispersion. J. Polit. Econ. 87, 851-858.

Rob, R., 1985. Equilibrium price distributions. Rev. Econ. Stud. 81, 1283-1308.

Rothschild, M., 1974. Searching for the lowest price when the distribution of prices is unknown. J. Polit. Econ. 82, 689-711.

Rudin, W., 1976. Principles of Mathematical Analysis. McGraw-Hill, New York.

Sargent, T.J., 1987. Dynamic Macroeconomic Theory. Harvard University Press, Cambridge, Mass.

Stahl, D.O., 1989. Oligopolistic pricing with sequential consumer search. Amer. Econ. Rev. 79, 700-712.

Stahl, D.O., 1996. Oligopolistic pricing with heterogeneous consumer search. Int. Jour. Indust. Org. 14, 243-68.

Topkis, D.M., 1979. Equilibrium points in non-zero-sum n-person submodular games. SIAM J. Cont. Opt. 17, 773-787.

Topkis, D.M., 1998. Supermodularity and Complementarity. Princeton University Press, Princeton.

Varian, H.R., 1980. A model of sales. Amer. Econ. Rev. 70, 651-59.

Vives, X., 1990. Nash equilibrium with strategic complementarities. Journal of Mathematical Economics 19, 305-21.

Vives, X., 1999. Oligopoly Pricing. MIT Press, London.

Welland, R., 1964. On Kothe spaces. Trans. Amer. Math. Soc. 112, 267-277.

Wildenbeest, M.R., 2007. An empirical model of search with vertically differentiated products. Working paper.

Yu, H., Zhu, W., 2005. Large games with transformed summary statistics, Econ. Theory 26, 237-41.

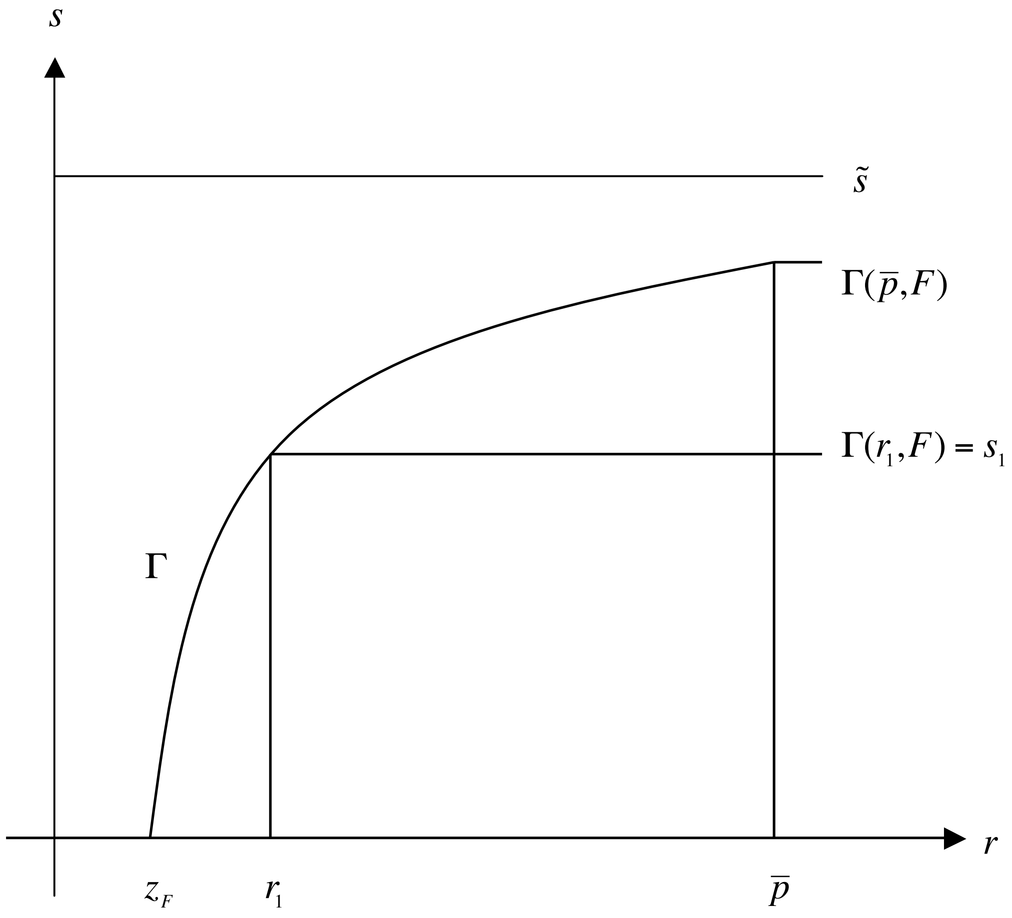

Figure 1. The marginal benefit of search.