

Exercice 217

Enoncé

Chercher un polynôme de degré 11: P tel que $P(X + 1) - P(X) = X^{10}$ en déduire $\sum_{k=0}^n k^{10}$

Commentaires

On cherche P sous la forme $\sum_{k=1}^{11} a_k X^k$, polynôme dont le terme constant est nul car si P convient il en est de même des polynômes $P + c$

On identifie les coefficients de $P(X + 1) - P(X)$ de degré $k = 0..10$ avec ceux de X^{10} , ce qui fait 11 équations à 11 inconnues.

$$\sum_{k=0}^n k^{10} = \sum_{k=0}^n P(k + 1) - P(k) = P(n + 1) - P(0) = P(n + 1)$$

Commandes MAPLE

Voir page suivante

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> restart;

> P:=x->sum(a[k]*x^k,k=1..11);

$$P := x \rightarrow \sum_{k=1}^{11} a_k x^k$$

> Q:=sort(expand(P(x+1)-P(x)));

$$\begin{aligned} Q := & 11 x^{10} a_{11} + 10 x^9 a_{10} + 55 x^9 a_{11} + 9 x^8 a_9 + 45 x^8 a_{10} + 165 x^8 a_{11} + 36 x^7 a_9 + 120 x^7 a_{10} + 330 x^7 a_{11} + 8 x^7 a_8 + 84 x^6 a_9 + 210 x^6 a_{10} + 462 x^6 a_{11} + 7 x^6 a_7 + 28 x^6 a_8 \\ & + 126 x^5 a_9 + 252 x^5 a_{10} + 462 x^5 a_{11} + 6 x^5 a_6 + 21 x^5 a_7 + 56 x^5 a_8 + 126 x^4 a_9 + 210 x^4 a_{10} + 330 x^4 a_{11} + 5 x^4 a_5 + 15 x^4 a_6 + 35 x^4 a_7 + 70 x^4 a_8 + 84 x^3 a_9 + 120 x^3 a_{10} \\ & + 165 x^3 a_{11} + 4 x^3 a_4 + 10 x^3 a_5 + 20 x^3 a_6 + 35 x^3 a_7 + 56 x^3 a_8 + 36 x^2 a_9 + 45 x^2 a_{10} + 55 x^2 a_{11} + 3 x^2 a_3 + 6 x^2 a_4 + 10 x^2 a_5 + 15 x^2 a_6 + 21 x^2 a_7 + 28 x^2 a_8 + 9 x a_9 \\ & + 10 x a_{10} + 11 x a_{11} + 2 x a_2 + 3 x a_3 + 4 x a_4 + 5 x a_5 + 6 x a_6 + 7 x a_7 + 8 x a_8 + a_9 + a_{10} + a_{11} + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_1 \end{aligned}$$

> eq:=k->coeff(Q,x,k);

$$eq := k \rightarrow \text{coeff}(Q, x, k)$$

> S:=seq(eq(k)=0,k=0..9),eq(10)=1;

$$\begin{aligned} S := & a_9 + a_{10} + a_{11} + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_1 = 0, 9 a_9 + 10 a_{10} + 11 a_{11} + 2 a_2 + 3 a_3 + 4 a_4 + 5 a_5 + 6 a_6 + 7 a_7 + 8 a_8 = 0, \\ & 36 a_9 + 45 a_{10} + 55 a_{11} + 3 a_3 + 6 a_4 + 10 a_5 + 15 a_6 + 21 a_7 + 28 a_8 = 0, 84 a_9 + 120 a_{10} + 165 a_{11} + 4 a_4 + 10 a_5 + 20 a_6 + 35 a_7 + 56 a_8 = 0, \\ & 126 a_9 + 210 a_{10} + 330 a_{11} + 5 a_5 + 15 a_6 + 35 a_7 + 70 a_8 = 0, 126 a_9 + 252 a_{10} + 462 a_{11} + 6 a_6 + 21 a_7 + 56 a_8 = 0, 84 a_9 + 210 a_{10} + 462 a_{11} + 7 a_7 + 28 a_8 = 0, \\ & 36 a_9 + 120 a_{10} + 330 a_{11} + 8 a_8 = 0, 9 a_9 + 45 a_{10} + 165 a_{11} = 0, 10 a_{10} + 55 a_{11} = 0, 11 a_{11} = 1 \end{aligned}$$

> res:=solve({S},{seq(a[k],k=1..11)});

$$res := \left\{ a_6 = 0, a_9 = \frac{5}{6}, a_{10} = \frac{-1}{2}, a_{11} = \frac{1}{11}, a_8 = 0, a_7 = -1, a_5 = 1, a_2 = 0, a_1 = \frac{5}{66}, a_3 = \frac{-1}{2}, a_4 = 0 \right\}$$

> assign(res);

> factor(P(x));

$$\frac{1}{66} x(x-1)(2x-1)(x^2-x-1)(3x^6-9x^5+2x^4+11x^3+3x^2-10x-5)$$

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