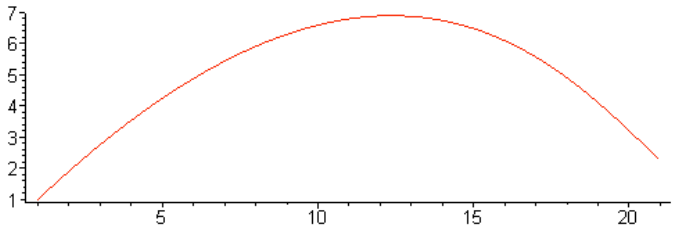
Exercice 1 Couple de suites récurrentes ?

```
> restart:
> u[0]:=1:
> v[0]:=1:
> N:=100:
> for k from 1 to N do u[k]:=u[k-1]+v[k-1]/k: v[k]:=u[k-1]/k+0.9*v[k-1] od:
> L:=[seq([u[p],v[p]],p=0..N)]:
> plot(L,scaling=CONSTRAINED);
```




```
> NL:=[seq(q*v[q]/u[q],q=0..N)];
```

$$NL := [0, 0.9500000000, 1.837288136, 2.664446367, 3.434703665, 4.151144125, 4.816736714, 5.434340513, 6.006705866, 6.536474678, 7.026180760, 7.478250414, 7.895003344, 8.278653901, 8.631312624, 8.954988112, 9.251589135, 9.522926930, 9.770717851, 9.996585989, 10.20206612, 10.38860664, 10.55757269, 10.71024925, 10.84784437, 10.97149236, 11.08225701, 11.18113478, 11.26905800, 11.34689796, 11.41546804, 11.47552667, 11.52778027, 11.57288613, 11.61145515, 11.64405453, 11.67121031, 11.69340989, 11.71110438, 11.72471084, 11.73461452, 11.74117085, 11.74470745, 11.74552597, 11.74390387, 11.74009608, 11.73433659, 11.72683991, 11.71780251, 11.70740408, 11.69580882, 11.68316659, 11.66961393, 11.65527520, 11.64026336, 11.62468102, 11.60862117, 11.59216795, 11.57539738, 11.55837804, 11.54117164, 11.52383365, 11.50641377, 11.48895646, 11.47150137, 11.45408375, 11.43673488, 11.41948236, 11.40235049, 11.38536056, 11.36853110, 11.35187818, 11.33541561, 11.31915515, 11.30310674, 11.28727865, 11.27167768, 11.25630926, 11.24117763, 11.22628594, 11.21163639, 11.19723031, 11.18306828, 11.16915021, 11.15547538, 11.14204255, 11.12885006, 11.11589580, 11.10317733, 11.09069189, 11.07843650, 11.06640792, 11.05460275, 11.04301743, 11.03164827, 11.02049148, 11.00954319, 10.99879950, 10.98825644, 10.97791003, 10.96775629]$$

Exercice 2 Uneéquation différentielle du premier ordre

```
> restart:
> eq:=2*x*y(x)-x^2*diff(y(x),x)=x^2;
```

$$eq := 2\,x\,y(x) - x^2\left(\frac{d}{dx}y(x)\right) = x^2$$

```
> dsolve({eq,y(1)=2},{y(x)});
```

$$y(x) = x + x^2$$

> **# le raccord n'a pas été étudié en 0, pour x<0 on a y(x)=x+Bx^2#**

Exercice 3 Une famille de solutions d'une équation différentielle du premier ordre

> **restart:**

> **eq:=(1-x^2)*diff(y(x),x)-x*y(x)=1;**

$$eq := (1 - x^2)\left(\frac{d}{dx}y(x)\right) - x\,y(x) = 1$$

> **r:=dsolve({eq,y(0)=m},{y(x)});**

$$r := y(x) = -\frac{\sqrt{(x-1)(x+1)}\,\ln(x + \sqrt{-1+x^2})}{(x-1)(x+1)} - \frac{\left(-m - \frac{\pi}{2}\right)I}{\sqrt{x-1}\,\sqrt{x+1}}$$

> **f:=unapply(Re(rhs(r)),m);**

$$f := m \to \Re\left(-\frac{\sqrt{(x-1)(x+1)}\,\ln(x + \sqrt{-1+x^2})}{(x-1)(x+1)} - \frac{\left(-m - \frac{\pi}{2}\right)I}{\sqrt{x-1}\,\sqrt{x+1}}\right)$$

> **C:=m->plot(f(m),x=0..1,y=-5..5);**

$$C := m \to \mathrm{plot}(f(m), x = 0 .. 1, y = -5 .. 5)$$

> **with(plots):**

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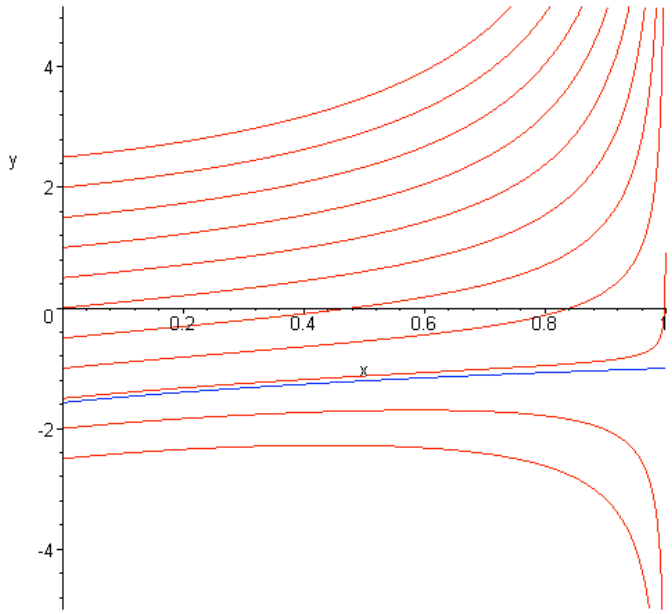
Warning, the name changecoords has been redefined
```

> **NC:=plot(f(-Pi/2),x=0..1, y=-5..5, color=blue): # courbe séparatrice en bleu#**

> **display([seq(C(q/2),q=-5..5), NC]);**



Exercice 4: Solution périodique d'une équation différentielle

> **restart:**

> **f:=x->sin(x)^2:**

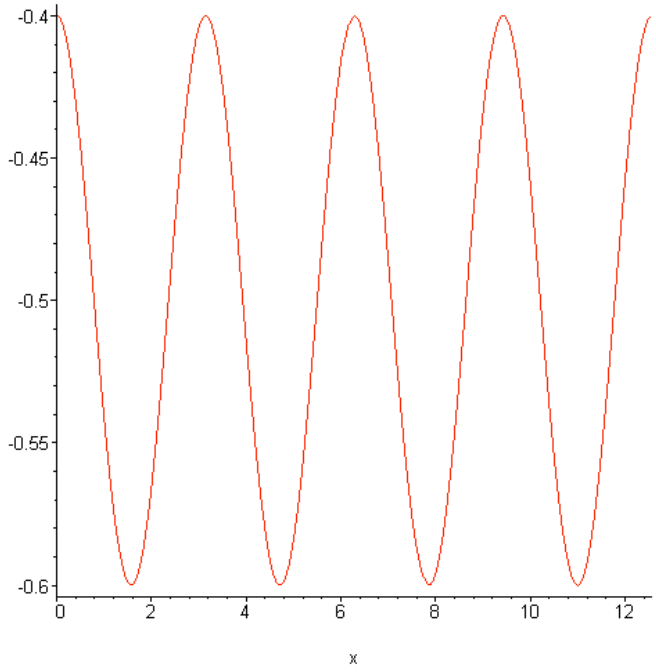
> **eq:=diff(y(x),x,x)-y(x)=f(x);**

$$eq := \left(\frac{d^2}{dx^2} y(x)\right) - y(x) = \sin(x)^2$$

> **r:=dsolve({eq, y(0)=y(2*Pi), D(y)(0)=D(y)(2*Pi)},{y(x)});**

$$r := y(x) = -\frac{3}{5} + \frac{1}{5}\cos(x)^2$$

> **plot(rhs(r),x=0..4*Pi);**



Exercie 5 Un raccord de deux solutions d'une équation différentielle du premier ordre

> **restart:**

> **eq1:=x*diff(y(x),x)+y(x)=x^2*cos(x);**

$$eq1 := x\left(\frac{d}{dx} y(x)\right) + y(x) = x^2 \cos(x)$$

> **f1:=unapply(rhs(dsolve({eq1,y(Pi/2)=a},{y(x)})),x);**

$$f1 := x \to \frac{x^2 \sin(x) - 2\sin(x) + 2x\cos(x) + \frac{a\pi}{2} - \frac{\pi^2}{4} + 2}{x}$$

> **eq2:=-x*diff(y(x),x)+y(x)=x^2*cos(x);**

$$eq2 := -x\left(\frac{d}{dx} y(x)\right) + y(x) = x^2 \cos(x)$$

> **f2:=unapply(rhs(dsolve({eq2,y(-Pi/2)=b},{y(x)})),x);**

$$f2 := x \to -x\sin(x) + \frac{2x\left(-b - \frac{\pi}{2}\right)}{\pi}$$

> **g:=expand(x*(f1(x)-f2(x)));**

$$g := 2\,x^2\sin(x) - 2\sin(x) + 2\,x\cos(x) + \frac{a\,\pi}{2} - \frac{\pi^2}{4} + 2 + \frac{2\,x^2\,b}{\pi} + x^2$$

> **s:=series(g,x,4);**

$$s := \left(\frac{1}{2}a\,\pi - \frac{1}{4}\pi^2 + 2\right) + \left(\frac{2\,b}{\pi} + 1\right)x^2 + \frac{4}{3}x^3 + O(x^4)$$

> **P:=convert(s,polynom);**

$$P := \frac{a\,\pi}{2} - \frac{\pi^2}{4} + 2 + \left(\frac{2\,b}{\pi} + 1\right)x^2 + \frac{4\,x^3}{3}$$

> **rs:=solve({coeff(P,x,0)=0, coeff(P,x,2)=0},{a,b});**

$$rs := \left\{a = \frac{\pi^2 - 8}{2\,\pi},\ b = -\frac{\pi}{2}\right\}$$

> **assign(rs);**

> **with(plots):**

```
Warning, the name changecoords has been redefined
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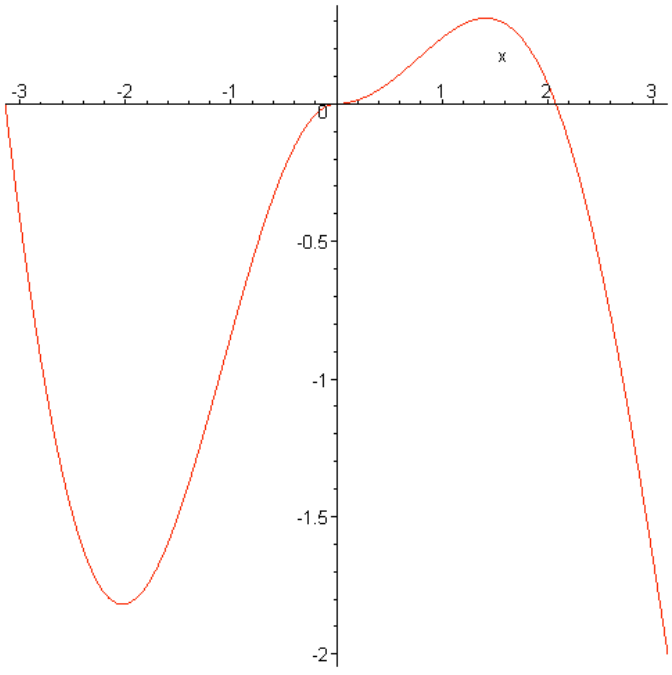
> **C1:=plot(f1(x), x=0..Pi): C2:=plot(f2(x),x=-Pi..0):**

> **display([C1,C2]);**



Exercice 6 Résolution approchée

> **restart:**

> **eq:=m-> 2*(1-x^2)*diff(y(x),x,x)-2*m*diff(y(x),x)+y(x)=0;**

$$eq := m \to 2\,(1 - x^2)\left(\frac{\partial^2}{\partial x\,\partial x}\mathrm{y}(x)\right) - 2\,m\left(\frac{d}{dx}\mathrm{y}(x)\right) + \mathrm{y}(x) = 0$$

> **f:=m->dsolve({eq(m),y(0)=-1, D(y)(0)=0},{y(x)}, type=numeric);**

$$f := m \to \mathrm{dsolve}(\{\mathrm{y}(0) = -1, \mathrm{D}(y)(0) = 0, \mathrm{eq}(m)\}, \{\mathrm{y}(x)\}, type = numeric)$$

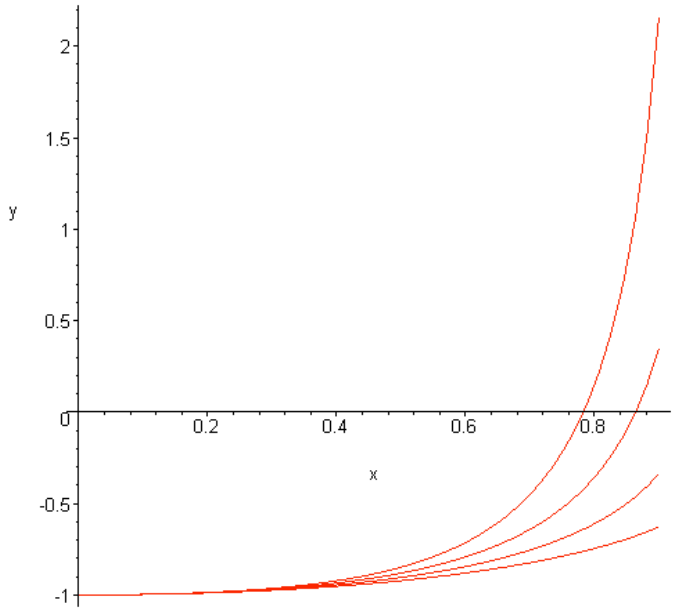
> **with(plots):**

```
Warning, the name changecoords has been redefined
```

> **C:=m->odeplot(f(m), [x,y(x)],0..0.9);**

$$C := m \rightarrow \text{odeplot}(f(m), [x, y(x)], 0 .. 0.9)$$

> **display([C(1),C(2),C(3),C(4)]);**



Exercice 7 Methode de la variation de la constante

> **restart:**

> **eq:=r->(1-x)*(9-x)*diff(r,x,x)+(3*x-19)*diff(r,x)-8*r;**

$$eq := r \rightarrow (1-x)(9-x)\left(\frac{\partial^2}{\partial x\, \partial x} r\right) + (3x-19)\left(\frac{d}{dx} r\right) - 8r$$

> **P:=sort(expand(eq(5*x^2+a*x+b)),x);**

$$P := -290x - 5ax + 90 - 19a - 8b$$

> **r:=solve({coeff(P,x,1)=0, coeff(P,x,0)=0},{a,b});**

$$r := \{a = -58, b = 149\}$$

> **assign(r);**

> **neq:=sort(expand(eq((5*x^2+a*x+b)*y(x))),x)=0;**

$$neq := 5\left(\frac{d^2}{dx^2} y(x)\right)x^4 - 108\left(\frac{d^2}{dx^2} y(x)\right)x^3 + 35\left(\frac{d}{dx} y(x)\right)x^3 - 585\left(\frac{d}{dx} y(x)\right)x^2 + 774\left(\frac{d^2}{dx^2} y(x)\right)x^2 + 2889\left(\frac{d}{dx} y(x)\right)x - 2012\left(\frac{d^2}{dx^2} y(x)\right)x$$
$$- 3875\left(\frac{d}{dx} y(x)\right) + 1341\left(\frac{d^2}{dx^2} y(x)\right) = 0$$

> **nneq:=subs(diff(y(x),x)=z(x),neq);**

$$nneq :=$$

$$5\left(\frac{d}{dx}z(x)\right)x^4 - 108\left(\frac{d}{dx}z(x)\right)x^3 + 35\,z(x)\,x^3 - 585\,z(x)\,x^2 + 774\left(\frac{d}{dx}z(x)\right)x^2 + 2889\,z(x)\,x - 2012\left(\frac{d}{dx}z(x)\right)x - 3875\,z(x) + 1341\left(\frac{d}{dx}z(x)\right) = 0$$

> **rz:=subs(_C1=c,rhs(op(dsolve({nneq},{z(x)}))));**

$$rz := \frac{c}{(x-1)^2\,(5x^2 - 58x + 149)\quad(x-9)^2}$$

> **f:=unapply(int(rz,x),x);**

$$f := x \to \frac{1}{73728}\,\frac{c}{x-1} - \frac{1}{65536}\,c\ln(x-1) + \frac{1}{65536}\,c\ln(x-9) + \frac{5}{9216}\,\frac{c\,x}{5x^2 - 58x + 149} - \frac{61}{18432}\,\frac{c}{5x^2 - 58x + 149}$$

> **g:=x->(f(x)+d)*(5*x^2+a*x+b);**

$$g := x \to (f(x) + d)\,(5x^2 + a\,x + b)$$

> **g(x);**

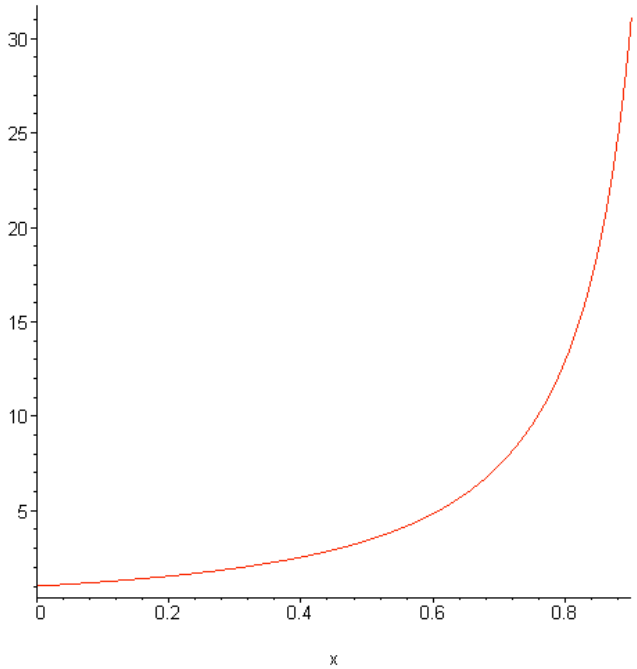
$$\left(\frac{c}{73728\,(x-1)} - \frac{1}{65536}\,c\ln(x-1) + \frac{1}{65536}\,c\ln(x-9) + \frac{5\,c\,x}{9216\,(5x^2 - 58x + 149)} - \frac{61\,c}{18432\,(5x^2 - 58x + 149)} + d\right)(5x^2 - 58x + 149)$$

> **nr:=solve({g(0)=1,D(g)(0)=2},{c,d});**

$$nr := \left\{d = -\frac{221}{2048} + \frac{801}{8192}\ln(3),\ c = -3204\right\}$$

> **assign(nr);**

> **plot(g(x),x=0..0.9);**



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