

## Mathematical Encounters\*

by Ihor Charischak

Stevens Institute of Technology, Hoboken, New Jersey

Contrary to popular opinion children come to our mathematics classrooms with plenty of ideas about mathematics. Unfortunately, not all of them are accurate. To illustrate what I mean, here is a conversation I had with Stewart, an average 8th grade student during a mathematics class. (I'm the T in this conversation and S is Stewart.)

T: What does  $4/8$  mean?

S: It means 4 out of 8. 4 parts out of 8. If you cut a pizza into 8 slices, 4 of them would make  $4/8$ .

T: What else can  $4/8$  equal?

S: (after some thought)  $1/2$ !

T: Good. Now, can you divide a larger number into a smaller number?

S: No.

T: You mean you can't do this:  $8 \mid 4$ ?

S: Oh, yeh, you can do that. That's equal to 2.

T: Wait a minute. Is that the answer to  $4 \mid 8$  or  $8 \mid 4$ ?

S: Doesn't matter. Either one. They are both equal to 2.

T: Really? But what if you have \$4 shared among 8 people, how much would each person have?

S: 2 dollars.

T: Well, let's think about that for the second. If 8 people each had two dollars, would they collectively have \$4?

S: No. (Student writes on paper  $\$4 \mid 8$  then crosses it out and writes  $8 \mid \$4$ .) Oh I got it! You do  $8 \mid \$4.00$ . (He proceeds to do the division.) The answer is 50 cents.

T: But isn't that a bigger number going into a smaller one?

S: No, it isn't. 400 is bigger than 8!

For a moment, I stopped and thought about what I should do or say next, but nothing of consequence came to mind. And since I was concerned that the teachable moment was slipping away, I just responded with the obvious question.

T: But isn't 4.00 the same as 4.0? And isn't 4.0 the same as 4?

S: Yeh...I guess so.

T: Then you are dividing a smaller number by a larger number!

S: Ok. If you say so.

End of teachable moment. Stewart didn't get it. Mercifully, the bell rang.

It was clear to me that just telling him did not work. What I was asking him to do was to look at some familiar ideas in a new, related way and try to make sense of how the various pieces fit together. He resisted because he was not comfortable with this kind of thinking, especially in math class.

Stewart's beliefs about division were interesting. He made it very clear that he believes that you can only divide a smaller number into a bigger number. However, sometimes the division problems are written "incorrectly" (like when the smaller number is in the dividend position.) When this happens, you can do one of

*\*This paper will be published as a part of the proceedings for the 10th Annual International Conference of Computers and Technology in Education, Boston, MA, March, 1993.*

two things. Either ignore the reversal and divide the smaller number into the bigger one, or just simply make the dividend *bigger* than the divisor by appending two or more zeros (with a decimal point thrown in.) This does not play havoc with his initial notion about division, because when you divide 8 into 4.00, what you are really doing is dividing 8 into 400. The placement of the decimal points is the window dressing that somehow justifies this "hocus pocus". Now you would think that this would cause Stewart some cognitive discomfort and would motivate him to try to understand how these ideas are connected. But, alas, Stewart is perfectly comfortable living with this ambiguity, as long as he knows what his teacher expects of him on the next test. Here the teacher and student make an agreement (a compromise) that if Stewart can perform to a certain level on the test, then he "knows" the material. With this understanding the student heaves a sigh of relief knowing that he won't be responsible for *really* knowing what is going on.

So how can I get the Stewarts of this world to go after real learning, rather than these compromises? One approach is to create situations where students have to confront and make sense out of situations which may be counter to their beliefs. Howard Gardner, in his book, *The Unschooled Mind*, calls these situations "Christopherian Encounters" (named after the explorer) "where students' earlier models or misconceptions are brought into sharp focus because of an experience that directly challenges the viability of that model."<sup>1</sup> Encounters should be done in small groups and students should be encouraged to discuss, debate, and clarify ideas with group members. Research done by Clements and Nastasi suggests that activities that encourage cognitive conflict are beneficial to students' understanding of ideas. What follows are two examples of encounters involving the use of computers and calculators.

### **The "My Dear Aunt Sally" Encounter**

Students are told that there is a definite order that you follow when you are asked to do a multiple operation problem. For example, the answer to  $2+3 \times 4$  is not 20, but rather 14 which requires that you do the multiplication first. But why should that be? If you do this problem on some simple 4 function calculators, you will get an answer of 20. Why can't you just do the operations in the order they appear? The teacher can challenge the group to come up with a rationale for why 14 has to be the "right" answer. (By the way, this approach will upset a few students who have religiously accepted this rule and do not wish to reflect on it.) Some students will suggest using a computer. Using the program LogoWriter, the instruction PRINT  $2 + 3 * 4$  will produce 14. At this point, a discussion about why the computer follows the "My Dear Aunt Sally" rule and the calculator does not will most likely prove to be fruitful. A subsequent activity could be to have the students adopt their own order of operation rule and apply it to a set of "order of operations" problems in their textbook. It becomes fairly evident to most students within a relatively short period of time, that some kind of agreement is necessary. For example, students may conclude that the problem  $2+3 \times 4$  is ambiguous and that the textbook should not print such problems. To avoid the problem always use parentheses!

### **A Geometric Excursion: Spiros**

This geometric exploration which challenges the student's notion that mathematics is mostly about adherence to rigidly applied algorithms asks the participants to make

1. Gardner, Howard. *The Unschooled Mind - How Children Think and How Schools Should Teach*. USA: Basic Books, 1991. (pp. 157-158.)

sense of a collection of data. Start by choosing three whole numbers, say 3, 5, and 6. Next draw lines with those lengths perpendicular to each other. Using Logo commands and applying a scale factor of 10, the instructions become: FD 30 RT 90 FD 50 RT 90 FD 60 RT 90. Repeat the same pattern. Do this three more times.

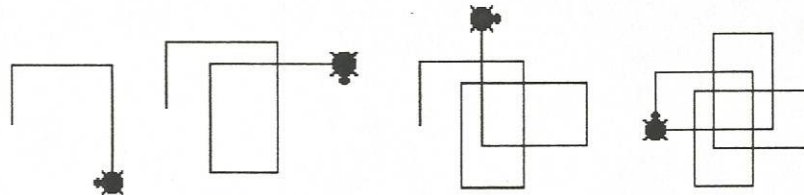


Figure 1

Notice that the pattern "completes" itself and the turtle is ready to draw the shape again. It turns out that if you follow this rule for any three positive whole numbers, the pattern will always "complete" itself after 4 repetitions. Using a Logo procedure (below) students working in small groups are asked to generate

```
to s :first :second :third
  repeat 4 [fd :first * 10 rt 90
            fd :second * 10 rt 90
            fd :third * 10 rt 90]
end
```

as many of these spirolateral patterns as possible. Next they classify them into categories and give these categories names. The students are then asked to come up

Gloop	Whimsy	Wumpus	Square
S 5 1 2 S 8 3 4 S 2 5 9	S 8 1 7 S 2 3 5 S 9 3 6	S 4 3 6 S 2 3 4 S 9 5 7	S 6 6 6 S 2 2 2 S 5 5 5

Figure 2

with conjectures about these classifications. For example, they look to see if they can predict what kind of shape will appear for any given set of three numbers.

### Bibliography

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