



Students' Interpretations

SUZANNE R. HARPER

SINCE GRAPHICAL DISPLAYS OF DATA ARE increasingly used in magazines, in newspapers, and on television to communicate relationships among numerical data, it is important to expose middle school students to various types of contexts where interpretation of graphical data is necessary. The NCTM (2000) concurs that students in grades 6–8 should “begin to compare the effectiveness of various types of displays in organizing the data for further analysis or in presenting the data clearly to an audience” (p. 49).

Friel, Curcio, and Bright (2001) distinguish between *graph comprehension*—“the graph readers’ abilities to derive meaning from graphs created by others or by themselves”—and behaviors that seem to demonstrate a presence of *graph sense* (p. 132). One characteristic of students’ having graph sense is of particular interest: “To recognize the components of graphs, the interrelationships among these components, and the effect of these components on the presentation of information in graphs” (p. 146). In *How to Lie with Statistics*, Huff (1954) speaks freely about the ease with which the creator of a graph can manipulate the viewer’s interpretation of the data. By asking students to answer questions where misleading graphs are used to display data, I hope to further understand students’ perceptions and interpretations of graphical displays of data (see **figs. 1, 2, and 3**). I invite you to try these problems with your students to see whether their performance reveals some interesting results.

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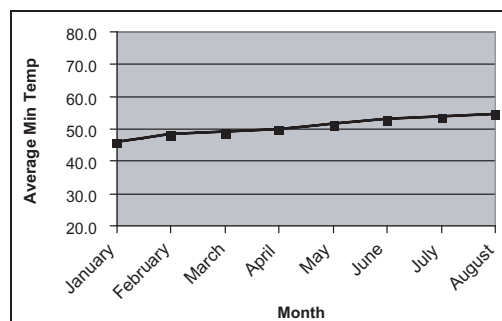
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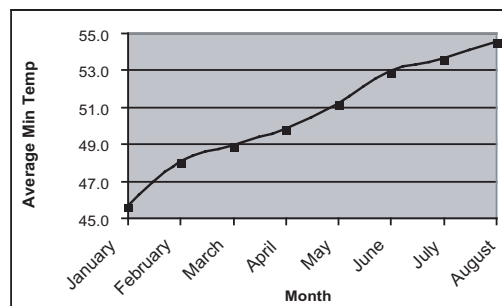
The average minimum monthly temperatures for San Francisco, California, have been recorded in the table, shown at right. The data have been correctly represented by both graphs A and B.

MONTH	°F
January	45.6
February	48.0
March	48.9
April	49.8
May	51.1
June	52.9
July	53.6
August	54.5

Graph A



Graph B



- Which graph would be best to help convince others that the average minimum temperature in San Francisco is much colder in January than in August? Explain your reason for making this selection.
- Why might people who thought that there was little difference between the average minimum temperature in January and August consider the graph you chose to be misleading?

Fig. 1 Problem 1: Average minimum temperature in San Francisco, California

tions of Misleading Graphs

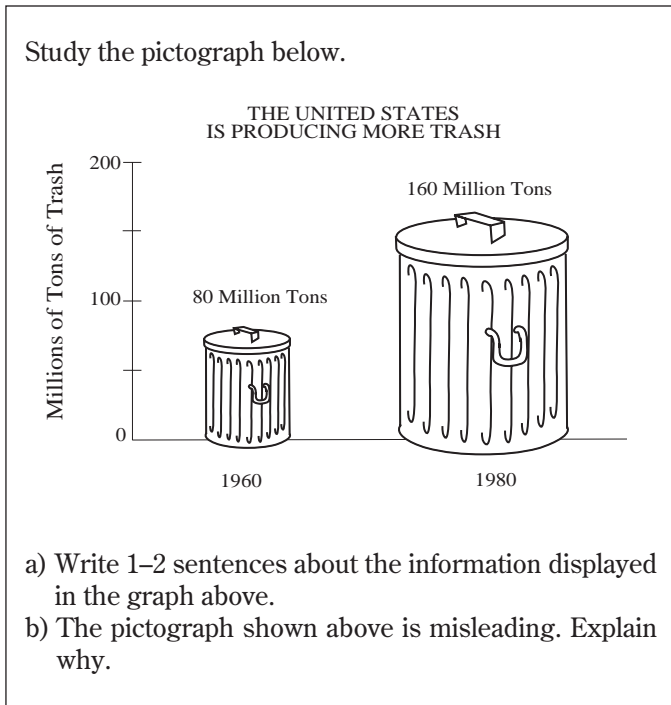


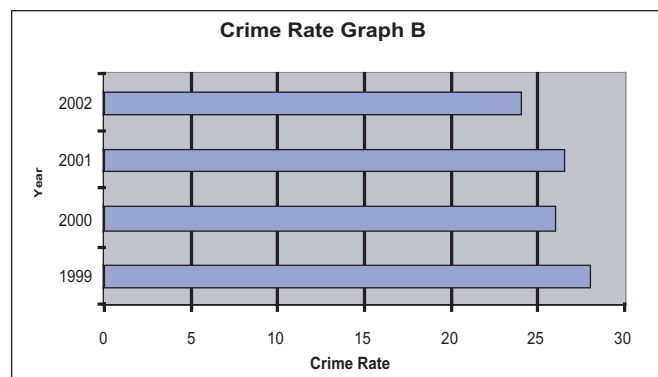
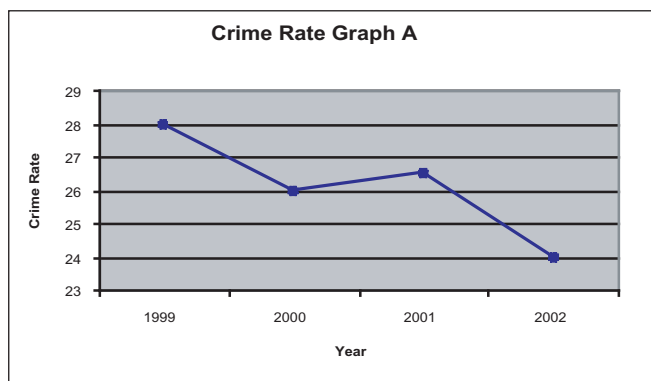
Fig. 2 Problem 2: United States trash production

Discussion of the Problems

THE SELECTED PROBLEMS FOCUS ON STATISTICS AND data analysis, primarily concerning the use and misuse of statistics in our everyday lives. Students were expected to use statistical concepts to analyze and communicate valid interpretations of the graphical and tabular forms of data from a variety of real-world contexts.

Although viewing data graphically allows the reader to see the trend of the data easily, the choice of scaling along the x - and y -axis of the graph could influence their interpretation of the data. For example, a question similar to problem 1 was administered on the 1996 National Assessment of Educational Progress (NAEP) eighth-grade exam (see **fig. 1**). Only 2 percent of these students answered the question correctly by identifying this information: (1) Graph B is the appropriate choice, since it appears to show a large increase between the smallest and largest data point; and (2) graph B is misleading since the y -axis scale exaggerates a relatively small increase. Of the students, 19 percent partially answered the question correctly by identifying graph B with an incomplete or partially correct explanation. The remaining students either incorrectly or minimally answered the question or omitted the test item altogether.

Mayor McCann is running for a second term as mayor against the challenger, Representative Royce. One of the hot topics of the campaign is crime prevention. Each graph displays the number of crimes per 1000 citizens for the four years of Mayor McCann's current term.



- a) What are the differences between the two graphs?
- b) If you were Representative Royce's campaign manager, which graph would you choose to use in the campaign? Why?
- c) If you were Mayor McCann's campaign manager, which graph would you choose to use in the campaign? Why?

Fig. 3 Problem 3: Crime rate of Metropolis

Problem 2 was also used on a NAEP eighth-grade exam in 1992 (see **fig. 2**). From the posted numerical values, we see that in 1960 the United States was producing half as much trash as it did in 1980. However, the volume of the trash-can icons does not appear to be in a proportion of 1:2 as the numerical data indicates. It appears that both the radius and the height of the 1960's trash can have doubled, causing the ratio of the volume of the cylinders to be 1:8. Using the NAEP grading rubric, students receive full credit for their solution if they indicate that (1) both width *and* height have been doubled; (2) the trash can in 1980 holds *much more* than twice the amount of the 1960 trash can; or (3) the ratio of the amount in the 1960's can to the amount in the 1980's can is less than 1:2. On the NAEP exam, only 8 percent of student responses were correct, 86 percent were incorrect, and 6 percent omitted the problem.

Although the first two problems were chosen to illustrate specific characteristics of misleading graphs, the third problem was selected to explore various reasons why certain groups of people would display data differently (see **fig. 3**). As Kosslyn (1994) describes, "A good graph forces the reader to see the information the designer wanted to convey" (p. 271). By having Mayor McCann's campaign manager use graph A, which shows a drastic drop in the crime rate, the desired effect to which Kosslyn refers is achieved.

Results of Students' Responses

THREE CLASSES WITH A TOTAL OF 59 SEVENTH-GRADE prealgebra students were asked to respond to the problems. (Summary tables characterizing the "correctness" of the students' answers are displayed in **tables 1, 2, and 3**.) All the students were taught by the same teacher who recently completed a three-week data analysis unit. The unit's objectives were (1) to analyze a set of data using and comparing combinations of measures of central tendency (mean, mode, median) and measures of spread (range, quartile, interquartile range) and describe how the inclusion or exclusion of outliers affects those measures; and (2) to read, create, and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs.

In problem 1, all but a single student (who omitted the question) correctly identified graph B as being an appropriate display to convince others that the average minimum temperature in San Francisco is much colder in January than in August. However, the vast majority of students did not provide valid reasoning for their selection or explain why graph B could be interpreted as misleading, which classified their responses as partially correct (see **table 1**). A typical response from this group of students reads, "On graph A, I really can't see any changes in temp so I wouldn't be able to tell the difference in temp between January and August. On graph B, you can really tell because it is spaced out and more clear." Even though the student described the physical characteristics of the graph, he failed

Table 1

PROBLEM 1	NUMBER OF STUDENTS	PERCENTAGE OF STUDENTS
Correct	7	11.9%
Partially Correct	34	57.6%
Minimally Correct	17	28.8%
Omitted	1	1.7%

Table 2

PROBLEM 2	NUMBER OF STUDENTS	PERCENTAGE OF STUDENTS
Correct	4	6.8%
Incorrect	49	83.0%
Omitted	6	10.2%

Table 3

PROBLEM 3	NUMBER AND PERCENTAGE OF STUDENTS WHO CHOSE GRAPH A	NUMBER AND PERCENTAGE OF STUDENTS WHO CHOSE GRAPH B	NUMBER AND PERCENTAGE OF STUDENTS WHO OMITTED QUESTION
Royce's Manager	24 (40.7%)	25 (42.4%)	10 (16.9%)
McCann's Manager	33 (55.9%)	11 (18.7%)	15 (25.4%)

to mention how the scale or range of numbers along the *y*-axis changed the appearance of the graph. Just over one-fourth of students (27.1%) responded that some aspect of the scale along the *y*-axis had changed. One student correctly reasoned why graph B might be considered misleading by responding, "There was little difference [between the temperatures]. They might think that graph B is misleading because it shows a big change." This student's explanation indicates his understanding of the numerical difference in temperature from January to August and the effects the different scales have along the *y*-axis.

In response to problem 2, 83 percent of students incorrectly justified why the graph was misleading (see **table 2**). Of these incorrect responses, only four mentioned the relative size of the trash cans. Sixteen students addressed either the scaling of the horizontal or vertical axes as being the graph's misleading attribute. One student concentrated on the numerical values along the *y*-axis, "It is misleading because there are no numbers in between the 0–100 and 100–200 to tell exact measurements." Another answer focused on the *x*-axis scaling, "This pictograph is misleading because it should have went by years like 1960, 1970, 1980 besides 1960 then 1980." A couple of students thought that the *type* of trash can was misleading readers to think that 80

million tons of trash would physically fit in that type of can. Only four students produced correct responses, such as “The picture of the trash can in 1960 is supposed to be half of the trash can in 1980, but it’s much more than half.” It was interesting to find that none of the students’ responses indicated they noticed that both width and height of the 1960 trash can had been doubled. The sample of seventh-grade student responses to both problems 1 and 2 remarkably paralleled those of the reported 1992 and 1996 NAEP results.

The responses to problem 3 were the most interesting to analyze. When the students were asked about which graph Representative Royce should use in his campaign, they responded with no clear favorite (see **table 3**). Approximately 41 percent favored graph A, 42 percent favored graph B, and 17 percent did not respond to the question. Of the students who selected graph A, the majority of explanations were based on the look of the graph: “It’s more professional and accurate,” “. . . easier to read,” “It has less amount of crime rate,” and “. . . shows the difference in the number.” In comparison, the responses of students who chose graph B were more likely to concentrate on the candidate’s campaign to win the election. Their reasons were primarily politically motivated: “We don’t want him [McCann] to win. And that [graph] doesn’t show much information”; “It shows the crime rate going up and needs to be fixed”; and “It doesn’t really look like the rate has gone down.” When the students were asked about Mayor McCann’s graph selection, fifteen students who preferred the line graph for Royce also chose to use the line graph for McCann, almost all citing the exact same physical reasons as before. However, of those students who responded to the question, 75 percent overwhelmingly selected graph A. Most students agreed that the crime rate “looks like it drops a lot more” in graph A, helping Mayor McCann secure his bid for reelection.


Implications for Teaching

ALTHOUGH MOST MIDDLE SCHOOL STUDENTS HAVE learned how to read and extract information from various types of graphs, it might not be obvious to them how particular graphs can be misleading or inaccurate. In this study, many students noticed the graphs’ axis scaling as a misleading feature; however, they were not as keen to observe pictorial embellishments or consider the source of the graph. A possible classroom idea would be to have your students construct various “misleading” graphs to communicate information they wish to convey on a particular issue. Students could discuss the different messages that each graph provides. Having students study graphs in current magazines and newspapers, like the problems described in this article, will not only help them become more “statistically literate” but also aware of the possible political agenda in some media sources. Finding information about the individuals who have created a graph and the target audience of the graph can lead to some insightful classroom discussions and conclusions.

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The author would like to recognize the seventh-grade students in Cincinnati, Ohio, who participated in this study and to express gratitude to their teacher for taking time out of her busy schedule to accommodate her. □



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